Geometrical Optics

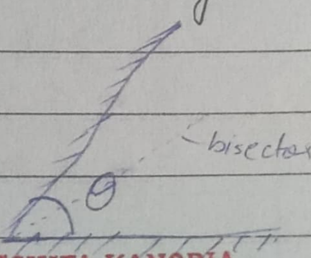
1. If incident ray $\rightarrow v, f, \lambda$, then
$\Rightarrow$ reflected ray $\rightarrow v, f, \lambda$
$\Rightarrow$ refracted ray $\rightarrow v', f, \lambda'$.
$v = f\lambda$

2. Laws of reflection - (i) incident ray, reflected ray and normal at the point of incidence, all lie in one plane (plane of incidence/ reflection).
(ii) $\angle i = \angle r$.

3. (i) $(\hat{I} \times \hat{N}) \cdot \hat{R} = (\hat{I} \times \hat{R}) \cdot \hat{N} = (\hat{N} \times \hat{R}) \cdot \hat{I} = 0$.
(ii) $(\hat{I} \times \hat{N}) = (\hat{R} \times \hat{N})$
(iii) $\hat{R} = \hat{I} - 2\hat{N}(\hat{I} \cdot \hat{N})$

4. During reflection, deviation $\delta = \pi - 2i$.
5. Real object $\Leftrightarrow$ Virtual image
Virtual object $\Leftrightarrow$ Real image.

6. If mirror (plane) is rotated through θ, the reflected ray rotates through 2θ in the same direction.
7. Field of view - the region in which the observer's eye must be present in order to view the image.
8. Time in clock = $A^H B^M C^S$
$\Rightarrow$ time in mirror image of clock = $11 - 59' - 60'' - A^H B^M C^S$.
(due to lateral inversion)
9. The minimum length of the plane mirror should be half the height of the person, to see a complete image of the person in the plane mirror.
10. If two plane mirrors are inclined to each other at an angle θ (number of images = n)
(i) If $\frac{360°}{\theta} = m$ (even) $\Rightarrow$ $n = m - 1$.



(ii) If $\frac{360°}{\theta} = m$ (odd)

If object is not on bisector ⇒ $n = m$.
If object is on bisector ⇒ $n = m - 1$.

Eye is most sensitive to yellow green colours (555 nm)

11. In ⊥ plane to mirror − $V_I = 2V_m - V_o$
In ∥ plane to mirror − $V_I = V_o$.

12. No. of images formed by two ∥ mirrors at a distance $d = \infty$.
Image positions = cross addition of d. from the 2 mirrors
If $i = 60°$ for a ray parallel to the principle axis of a concave mirror, then reflected ray passes through the pole (and not focus)

13. $f = \frac{R}{2}$

If θ → small ⇒ paraxial rays
θ → large ⇒ marginal rays

14. For concave mirror − $f = -ve$
convex mirror − $f = +ve$

15. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ —— For spherical mirrors

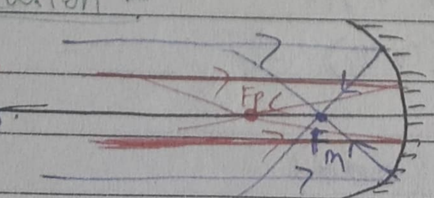
16. $P = \text{Power} = -\frac{1}{f\,(\text{in } m)}$

17. Lateral / Transverse / Linear Magnification − (m_T)
$m = -\frac{v}{u} = \frac{h_2}{h_1} = \frac{f}{f-u}$

$m = +ve$ ⇒ virtual image
$m = -ve$ ⇒ real image.
$|m| > 1$ ⇒ enlarged
$|m| < 1$ ⇒ diminished

For successive reflections,
$m = m_1 \times m_2 \times m_3 \ldots$

spherical abberration −
— = paraxial rays
— = marginal rays



18. **Longitudinal Magnification (m_L)** – (For objects placed along the principal axis)

$$m_L = \frac{I}{O} = \frac{v_2 - v_1}{u_2 - u_1}.$$

For small objects $m_L = \frac{dv}{du}$

$$m_L = -m_T^2$$

Superficial Magnification [For 2-D objects placed ⊥ to principal axis]

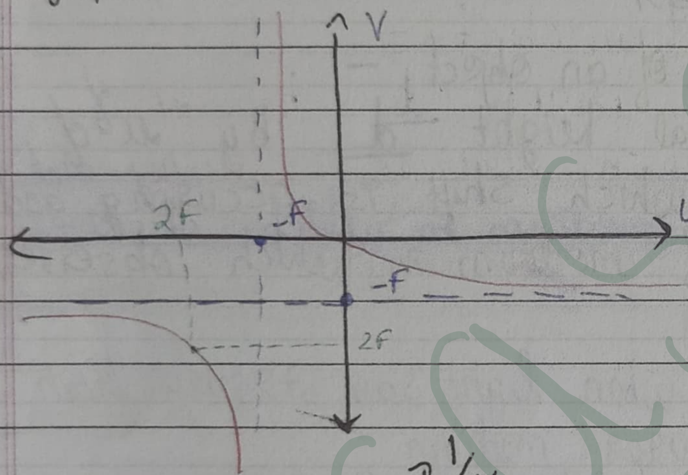
(h = height
w = width)

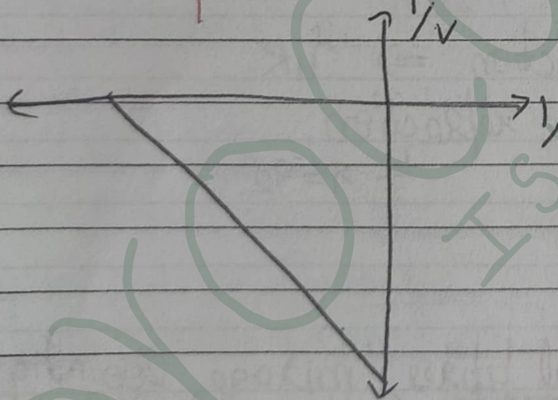
$$h_i = m h_o$$
$$w_i = m w_o$$

$(A = w \times h)$

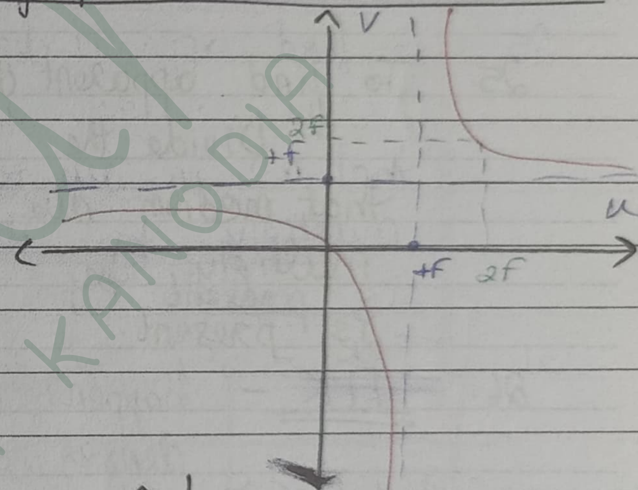
$$A_i = m^2 A_o.$$

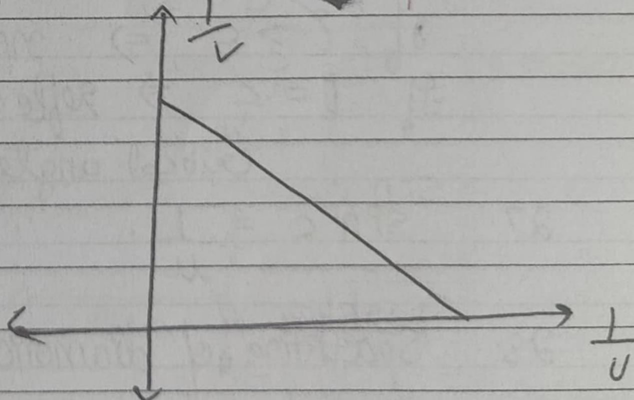
19. **Graph For Concave Mirror**





Graph For Convex Mirror



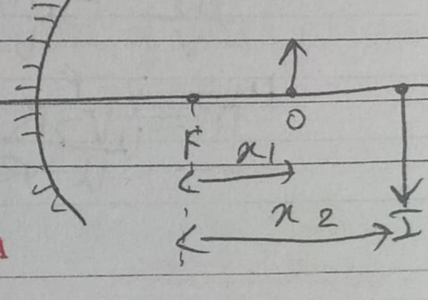


20. Image is smaller than object ⇒ convex mirror ⇒ car rear mirror
Image is larger than object ⇒ concave mirror ⇒ shaving mirror.

21.

$$x_1 x_2 = f^2$$

– Newton's formula



$n = \mu =$ refractive index.

22. Snell's Law – $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2 = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

$v = \frac{c}{\mu}$ ($n_1 \sin i = n_2 \sin r$)

23. $\delta = |i - r|$

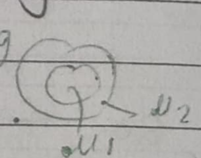
24. During refraction from glass (parallel) slab, emergent ray ∥ incident ray
(only when medium on both sides of the glass slab is same).
Lateral shift $(d) = \frac{t}{\cos r} \sin(i - r)$.

25. To find apparent depth of an object, – Divide the actual height d by μ of that medium due to which shift is occuring and multiply by μ of medium in which observer is present.

26. TIR – happens only when light ray travels from denser to rarer medium.
If $i > c \Rightarrow$ no refraction $\Rightarrow$ TIR
If $i = c \Rightarrow$ reflection + refraction
c: critical angle; $r = 90°$.

27. $\sin c = \frac{1}{\mu}$.

28. Sparkling of diamond, optical fibres, mirage, looming $\Rightarrow$ TIR.
μ of core is > μ of cladding ($\mu_1 > \mu_2$)



29. For refraction at spherical surfaces –
$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$ (when light travels from μ_1 to μ_2)

$m = \frac{v - R}{u - R} = \frac{v / n_2}{u / n_1}$.

In case of plane surfaces, $R = \infty$.

30. For lens,

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ — Lens formula

$\frac{1}{f} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ — Lens makers formula

$\left(\mu_r = \frac{\mu_{lens}}{\mu_{surrounding\ medium}}\right)$ (sm)

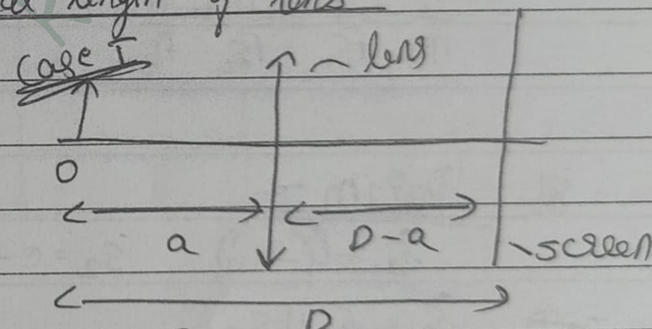
$f = -ve$ for concave lens
$f = +ve$ for convex lens.

$\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ⇒ positive for all convex lens
⇒ negative for all concave lens.

$(\mu_r - 1)$ ⇒ positive when ~~rarer~~ sm is rarer than lens
⇒ negative when sm is denser than lens.

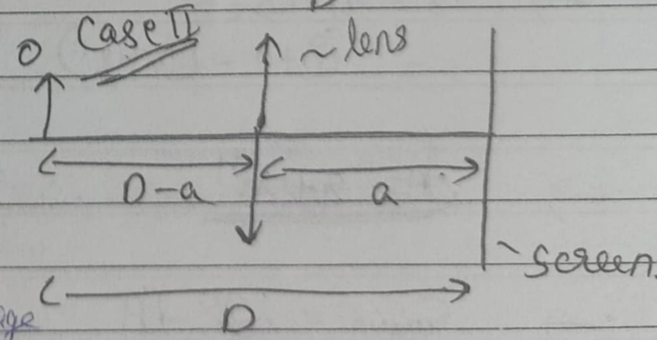
31. $m = \frac{v}{u} = \frac{f}{f+u}$.

32. Displacement method to find focal length of lens –



$a^2 - Da + Df = 0$

a will have one value if –
$D^2 - 4Df = 0$
⇒ $D = 4f$
⇒ $a = 2f$ $\left(a = \frac{D \pm \sqrt{D^2 - 4Df}}{2}\right)$



a will have 2 values if
$D^2 - 4Df > 0$
⇒ $D > 4f$

a will have no value for sharp image
$D^2 - 4Df < 0$
⇒ $D < 4f$.

$a_2 - a_1 = \sqrt{D^2 - 4Df}$ = difference in position of lens in two cases.

$m_I m_{II} = 1$. ⇒ $h_1 h_2 = h_0^2$

(A = area) $A_1 A_2 = A_0^2$ (for 2-D objects).

33. <u>Combination of lenses</u> –

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots \ldots \quad \text{(for lens in contact)}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \quad \text{(for converging lens separated by distance } d\text{)}$$

Position of equivalent lens from second lens (f_2)

$$= -\frac{dF}{f_1}.$$

34. <u>Combination of lens and mirror / silvering lens on one surface</u>
(resultant = <u>mirror</u> (always))

$$\frac{1}{F} = \frac{1}{f_m} - \frac{2}{f_l}.$$

35. <u>Power of lens</u> – $P = \dfrac{1}{f(m)}$ [unit of Power – Dioptre (D)]

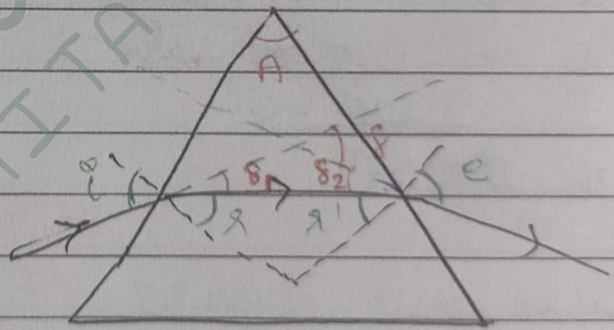
<u>For combination of lens,</u>

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \ldots \qquad P = P_1 + P_2 + P_3 + \ldots$$

36. <u>Prism</u> –

$\delta_1 = i - r$ $\quad \delta_2 = e - r'$

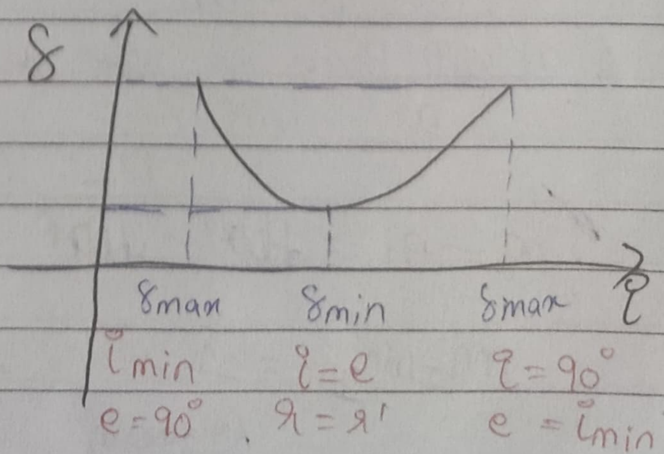
→ $\delta = \delta_1 + \delta_2$
$= i + e - (r + r')$



→ $\underline{A = r + r'}$ $\quad \therefore \underline{\delta = i + e - A}$

→ $\delta_{min} = 2i - A$

$i = \dfrac{\delta_{min} + A}{2}$, $\quad r = r' = \dfrac{A}{2}$.



→ $\mu = \dfrac{\sin\left(\dfrac{\delta_{min} + A}{2}\right)}{\sin \dfrac{A}{2}}$

→ If angles are small $(A \to 0$ or $A \leq 10^\circ)$

$$\mu = \frac{\frac{\delta_{min} + A}{2}}{\frac{A}{2}} \Rightarrow \frac{\delta_{min} + A}{A} = \mu.$$

$$\mu_r = \frac{\mu_{prism}}{\mu_{surrounding}}$$

→ $\delta_{min} = (\mu_r - 1) A$

37. **Dispersion of Light** – $\mu = a + \frac{b}{\lambda^2}$

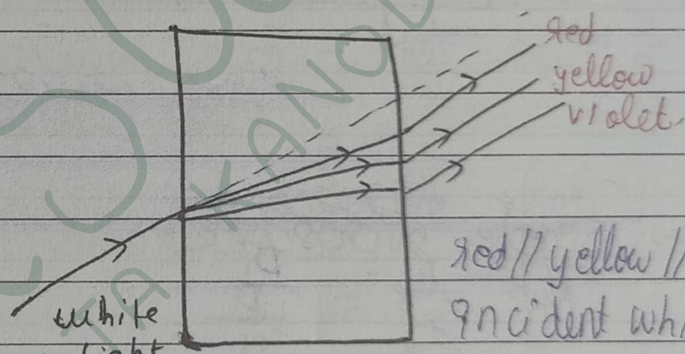
$\lambda_{red} = max$, $\lambda_{violet} = min$

$\therefore \mu_{violet} = max$, $\mu_{red} = min$

→ $\theta = \delta_v - \delta_r$. ($\theta$ = angle of dispersion)

→ For small angle prism,

$$\theta = (\delta_v - \delta_r)$$
$$= (\mu_v - \mu_r) A.$$



red // yellow // violet // incident white light
$\therefore \theta = 0^\circ$.

→ (ω = dispersive power of a medium)

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y}.$$

refractivity $= \mu - 1$.

38. **Dispersion without deviation (Direct combination vision)**

$\delta = 0$

$$\Rightarrow (\mu_y - 1) A = (\mu_y' - 1) A'.$$

Deviation without dispersion (Achromatic combination)

$\theta = 0$

$$\Rightarrow (\mu_v - \mu_r) A = (\mu_v' - \mu_r') A'.$$

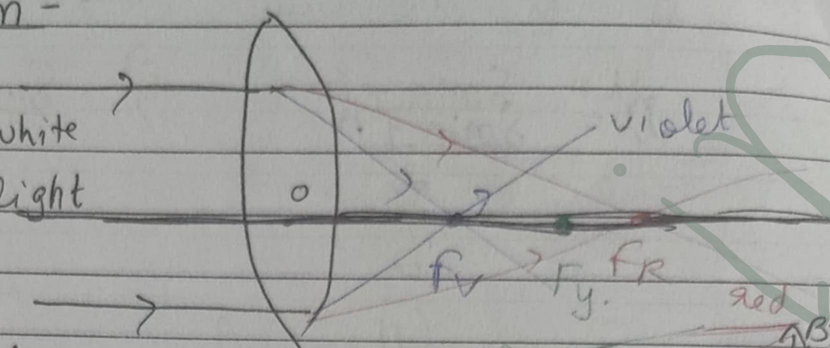
MP = magnifying power.

D = Least distance of distinct vision for a normal eye = 25 cm.

39. Chromatic Aberration -

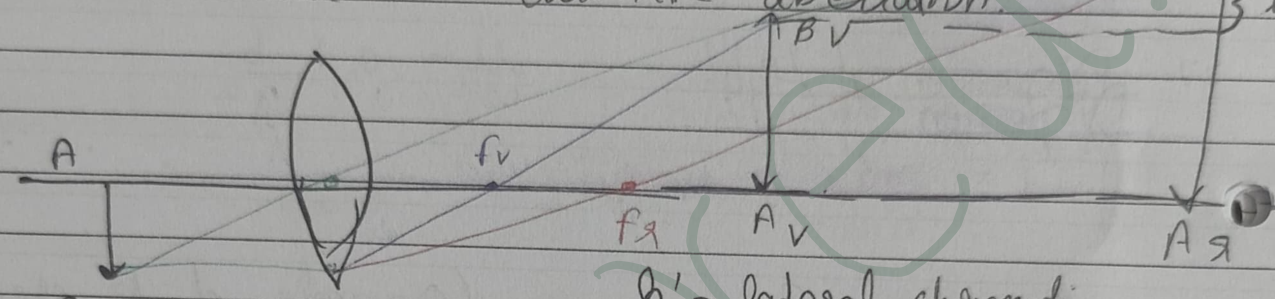
~~Longitud~~

→ $f_v f_R = f_y^2$



→ $\underbrace{f_v - f_R}_{\text{Longitudinal/Axial chromatic aberration}} = \omega f_y$.

→



h' = lateral chromatic aberation.

→ Condition for achromatism -

Two lens - one concave
- other convex

$\frac{\omega_1}{f_{y_1}} + \frac{\omega_2}{f_{y_2}} = 0$.

40. Simple microscope -

$MP = \frac{\beta}{\alpha} = \frac{D}{u}$,

If image is at D,	image is at ∞, (relaxed eye condition)
$u_{min} = \frac{Df}{D+f}$	$u_{max} = f$
$MP_{max} = 1 + \frac{D}{f}$	$MP_{min} = \frac{D}{f}$.

41. Compound microscope - 2 lens - object lens
- eye lens (acts as simple microscope.)

$M_o = \frac{V_o}{-u_o}$.

L= length of microscope/ telescope

<u>If image at ∞,</u>

→ $v_e = f_e$

→ $M_e = \frac{D}{f_e}$

→ $M = M_o M_e$

$M = -\frac{V_o}{u_o} \times \frac{D}{f_e}$

→ $L = V_o + u_e$

$= V_o + f_e$

<u>Image at D</u>

$v_e = \frac{Df_e}{D+f_e}$

$M_e = 1 + \frac{D}{f_e}$.

$M = M_o M_e = -\frac{V_o}{u_o}\left(1 + \frac{D}{f_e}\right)$.

$L = V_o + u_e$

$= V_o + \frac{Df_e}{D+f_e}$.

42. <u>Astronomical telescope</u> – 2 lens – object lens
(used to see distant objects) – eye lens (acts as simple microscope)

→ $\therefore u_o = \infty$

$V_o = f_o$. (Intermediate image is formed in plane of f_o)

→ $f_o >> f_e$. (for telescope)

→ $MP = \frac{\beta}{\alpha} = -\frac{f_o}{u_e}$.

Normal adjustment

<u>If image at ∞,</u>

→ $v_e = f_e$

→ $MP = -\frac{f_o}{f_e}$

→ $L = f_o + v_e$

$= f_o + f_e$

<u>If image at D,</u>

$u_e = \frac{Df_e}{D+f_e}$

$MP = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

$L = f_o + v_e$

$= f_o + \frac{Df_e}{D+f_e}$.

a = aperture
l = lens

43. Cassegrain's telescope — $MP = -\frac{f_o}{f_e}$

44. Galilean telescope — $MP = \frac{f_o}{f_e}$ $L = f_o + f_e$. (put all values with sign).

45.

Compound Microscope	Astronomical Telescope
used to increase visual angle of near tiny object.	used to increase visual angle of distant large objects.
f_o, f_e, a_o, a_e = less	f_o, a_o = large f_e, a_e = short
MP does not change appreciably if l_o and l_e are interchanged.	MP becomes $1/m^2$ times of original value when l_o and l_e are interchanged.
MP is increased by decreasing both f_o and f_e.	MP increased by increasing f_o and decreasing f_e.

46. Camera —

→ $I \times S \times t$ = constant (= energy) I = intensity S = area t = time.

→ If S = fixed,
$I \times t$ = constant.

→ $S \propto a^2$ (a = aperture)

When I = fixed,
$t \times a^2$ = constant.

time of exposure $\propto \frac{1}{(\text{aperture})^2}$

f number (f-n) = $\frac{\text{focal length}}{\text{aperture}}$.

$\therefore$ f-n $\propto \frac{1}{\text{aperture}}$

$\therefore$ time of exposure $\propto (f-n)^2$.

47. Defects of Eye - when image is not formed on retina.

I Myopia/short-sightedness

→ image forms before retina
→ thickness of eye lens ↑es.
→ Far-point ⇒ the distance till which person can see
(x) clearly.

$$\boxed{f = -x}$$

→ corrected using concave lens.

II Hypermetropia/Long-sightedness

→ image forms beyond retina
→ thickness of eye lens ↓es.
→ corrected using convex lens.

Near point (x)

$$\boxed{\frac{1}{f} = \frac{1}{D} - \frac{1}{x}}$$

III Presbyopia - person has both myopia and hypermetropia.
- both concave and convex lens are used.

IV Astigmatism - eye cannot see objects in two orthogonal directions.
- removed by using cylindrical lens in a particular direction.

48. Rainbow
→ observer's back faces the sun.
→ primary rainbow - TIR happens once in rain drop.
secondary rainbow - TIR happens twice in rain drop.

→ In primary rainbow, R is on top and V is at the bottom (opposite in secondary rainbow).

($40°, 42° \rightarrow 1°$ rainbow $50°, 53° \rightarrow$ secondary rainbow)

49. **Scattering of light** - It involves the absorption of light by the molecules followed by its re-radiation in diff. directions.

→ When scattering particles are of size smaller than the wavelength of light (λ) —

Rayleigh's law ← $I \propto \frac{1}{\lambda^4}$

$\because \lambda_{blue} < \lambda_{red}$

$\therefore$ blue is scattered more.

→ If scattering particles are of size greater than the wavelength of light (eg - dust particles, water droplets) - Rayleigh's law is not applicable and all colours are scattered equally. Thus, clouds appear white.