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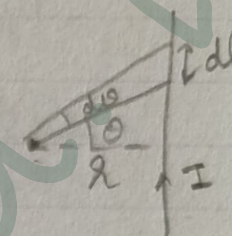
MAGNETIC EFFECT OF CURRENT, MAGNETISM

1. A current carrying conductor has a magnetic field and not an electric field around it.
2. Magnetic Field = flux density = magnetic induction ($\vec{B}$)
   unit = Tesla (T) or Weber/m^2
   CGS = gauss.
   $\}$ $1\,T = 10^4$ gauss.
3. $\vec{B}$ due to current carrying wire and due to magnet is frame independent. $\vec{B}$ due to moving charge is frame-dependent.
4. Biot-Savart Law -

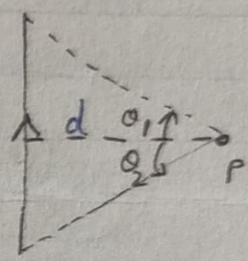
   $$\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}}. \qquad \left[d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\vec{r}}{r^3}\right].$$

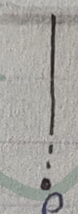
   μ_0 = permeability of free space = $4\pi \times 10^{-7}$ T-m/A.
   $I\,dl$ = current element.
   Right hand thumb rule [RHTR] for $d\vec{l} \times \vec{r}$.
5. For current carrying wire -

   $$\boxed{B = \frac{\mu_0}{4\pi}\frac{I}{d}(\sin\theta_1 + \sin\theta_2)}$$

   [d = ⊥ distance
   θ_1, θ_2 = measured from ⊥ in opposite sense]

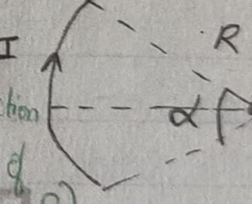
   $$\left[\ B_P = 0\ \right]$$

6. For current carrying circular arc -

   $$\boxed{B = \frac{\mu_0 I \alpha}{4\pi R}}.$$

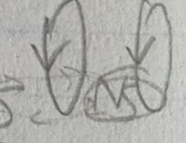
   (at centre)
   (curl the fingers in the direction of I, then thumb gives the direction of B).

   For coil ($n > 1$) ($\alpha = 2\pi$) - $\boxed{B = \frac{\mu_0 I n}{2R}}$
7. For Current Carrying Coil (At axial point) -

   $$\boxed{B_{axial} = \frac{\mu_0 I R^2 N}{2(R^2 + x^2)^{3/2}}}.$$

   (x = distance on the axis between centre of ring and axial point)
8. Loop and bar magnet have identical field lines
   ∴ Current carrying loop behaves like a bar magnet.
   Two coaxial loops carrying current in the same sense attract each other.

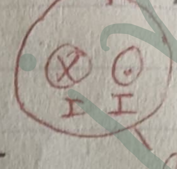
   

9. Helmholtz Coil Arrangement - two identical co-axial coils, the two coils are at a distance equal to the radius of the coils, planes are parallel, current direction is same in both coils.

10. Magnetic Field Lines – outside magnet from N to S
– inside magnet from S to N – lines never intersect
– magnetic field lines always exist in closed loops.

$\int \vec{E}\cdot d\vec{s} = q_{in}/\varepsilon_0$

11. Ampere's Circuital Law – $\boxed{\oint_{\text{closed loop}} \vec{B}\cdot d\vec{l} = \mu_0 I_{in}}$



12. Hollow Current Carrying Cylinder – $B_{out} = \dfrac{\mu_0 I}{2\pi r}$
$B_{in} = 0$.

$I_{net} = 0$
$\therefore B = 0$

Solid Current Carrying Cylinder – $(J = I/\pi R^2)$

$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2} \qquad \left[I_{in} = \frac{I r^2}{R^2}\right]$$

$$B_{out} = \frac{\mu_0 J R}{2r} = \frac{\mu_0 I}{2\pi r}.$$

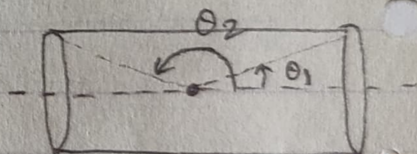
$B 2\pi r = I\mu_0$

13.

$$\boxed{B = \frac{\mu_0 J\, c_1 c_2}{2}}$$

14. Infinite Current Sheet – $\boxed{B = \dfrac{\mu_0 J_s}{2}}$ B is independent of distance from sheet.
B is always parallel to the plane of the sheet.

15. Solenoid – $\boxed{B = \dfrac{\mu_0 n I}{2}(\cos\theta_1 - \cos\theta_2)}$
n = no. of turns per unit length.



For ideal solenoid (at mid point) $(\theta_1 \to 0,\ \theta_2 \to \pi)$
$\boxed{B = \mu_0 n I}$

at end point $(\theta_1 \to \pi/2,\ \theta_2 \to \pi)$ $B = \dfrac{\mu_0 n I}{2}$.

For ideal solenoid ⇒ B outside the solenoid tends to 0.
B inside is uniform.

16. Toroid – (= endless solenoid) $\boxed{B_{in} = \mu_0 n I = \dfrac{\mu_0 N I}{2\pi R}}$ (Toroid = ring-shaped solenoid / solenoid joined end-to-end.)

B outside the volume of the toroid = 0.

B due to moving point charge

→ $\boxed{B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}}$ $\quad \left[\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}\right]$

Put value of q with proper sign.

→ $\boxed{\vec{F}_m = q(\vec{v}\times\vec{B})}$ $\quad [F_m = qvB\sin\theta]$.

18. ## Motion under effect of F_m

→ If $v = 0 \Rightarrow F_m = 0$

→ If $\vec{v} \parallel \vec{B} \Rightarrow F_m = 0. \Rightarrow$ uniform motion

→ If $\vec{v} \perp \vec{B} \Rightarrow F_m = qvB =$ constant. F is centripetal

$\therefore$ uniform circular motion. $a_c = \frac{F_m}{m} = \frac{qvB}{m}$.

$\boxed{R = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB}}$ $\quad \boxed{T = \frac{2\pi m}{qB}}$ $\quad (v = R\omega)$

Proton -	m	q
Deutron -	2m	q
α-particle -	4m	2q

→ If $\vec{v}$ is at θ to $\vec{B}$ → circular motion with uniform pitch (helical motion).

$F_m = qvB\sin\theta$

$R = \frac{mv\sin\theta}{qB}$ $\quad$ Pitch $= v_{\parallel}T = v\cos\theta\left(\frac{2\pi m}{qB}\right)$.

19. ## Lorentz force (L) -

$\vec{F}_L = \vec{F}_E + \vec{F}_m$

$= q\vec{E} + q(\vec{v}\times\vec{B})$.

$F_E >>> F_m$.

20. ## Velocity Selector (zero gravity region).

→ $\vec{E} \perp \vec{B} \perp \vec{v}$ and $F_e = F_m \Rightarrow$ undeviated motion

→ $E \parallel B \parallel v \Rightarrow F_m = 0$, $F_e = q\vec{E}$. ⇒ uniformly accelerated motion.

→ v is at θ to E and B $(E \parallel B)$ ⇒ helical path with non-uniform pitch. (zy plane = circular motion

x = uniformly accelerated).

$F = qvB\sin\theta$

$R = \frac{mv\sin\theta}{qB}$. $\quad a_x = \frac{qE}{m}$.

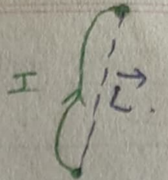
21. ## Force (F_m) On Current Carrying Wire -

$\boxed{d\vec{F} = I\, d\vec{l}\times\vec{B}}$

$\vec{F} = \int d\vec{F} = I\int d\vec{l}\times\vec{B}$

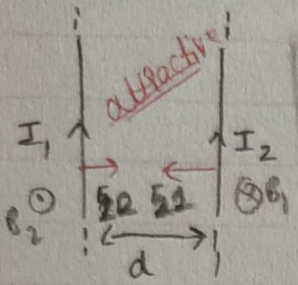
→ If $\vec{B}$ = constant ⇒ $\boxed{F = I\vec{L} \times \vec{B}}$

$\vec{L}$ = vector length

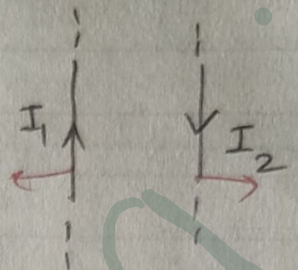


($\vec{L} = 0$ for closed loops ⇒ $F = 0$.)

On a straight current carrying wire, the magnetic force in a uniform B can be assumed to act at its mid-point.

22. 

$$\boxed{\frac{|\vec{F}_{21}|}{dl} = \frac{|\vec{F}_{12}|}{dl} = \frac{\mu_0 I_1 I_2}{2\pi d}}$$



($F = I\,dl\,B\sin\theta$, $I_2 \frac{\mu_0 I_1}{2\pi d}$, $\frac{F}{dl} = IB$)

23. Magnetic Dipole Moment ($\vec{M}$) –

$\boxed{\vec{M} = m\vec{l}}$ m = pole strength.

direction of $\vec{M}$ is always from $-m$ to $+m$ (S to N).

[unit $M = Am^2$
unit $m = A\text{-}m$]

→ For two very long bar magnets – $\boxed{F_m = \left(\frac{\mu_0}{4\pi}\right)\frac{m_1 m_2}{r^2}}$

→ For current carrying loop –

$\boxed{\vec{M} = NI\vec{A}}$

→ $\boxed{\frac{M}{L} = \frac{q}{2m}}$ (L = angular momentum, q = charge, M = dipole moment, m = mass)

$L = I\omega$.

[$I_{ring} = mR^2$, $I_{disc} = \frac{mR^2}{2}$, $I_{solid\ sphere} = \frac{2}{5}mR^2$, $I_{hollow\ sphere} = \frac{2}{3}mR^2$]

24. Atomic magnetism [Bohr's postulate ⇒ $L = mvr = \frac{nh}{2\pi}$]

$I = \frac{e}{T} = \frac{e\omega}{2\pi}$

$B_N = \frac{\mu_0 I}{2R} = \frac{\mu_0 e\omega}{4\pi R}$.

$M = I\vec{A} = \frac{evr}{2}$

$\boxed{\frac{M}{L} = \frac{e}{2m}}$ gyromagnetic ratio ($\therefore M = \frac{neh}{4\pi m}$).

$$\boxed{\begin{aligned}\vec{\tau} &= \vec{M}\times\vec{B}.\\ \vec{U} &= -\vec{M}\cdot\vec{B}.\end{aligned}}$$

$\theta = 0°$ ($U = -MB = min.$) $\Rightarrow$ stable equillibrium

$\theta = 180°$ ($U = MB = max.$) $\Rightarrow$ unstable equillibrium.

$\theta = 90°$ ($U = 0$) $\Rightarrow$ No equillibrium.

If net force is zero, then body rotates about an axis passing through the centre of mass and parallel to the direction of torque.

26. $\vec{B} = \dfrac{\vec{F}}{m}$.

→ Magnetic dipole formulae same as electric dipole.

Force on dipole in non-uniform B –

$$\boxed{|\vec{F_2}| = M_2\left(\frac{dB_1}{dr}\right)}$$

end on position = axial.

broad side on position = equatorial

→ Magnetic force on a short dipole M_2 due to M_1

$$\boxed{|F_2| = 6\,\frac{\mu_0}{4\pi}\,\frac{M_1 M_2}{r^4}.}$$

$B_1 = \dfrac{\mu_0}{4\pi}\dfrac{2M_1}{r^3}$ $\qquad \dfrac{dB_1}{dr} =$

$F = M_2\dfrac{dB_1}{dr}$

27. Cyclotron –

→ $R = \dfrac{mv}{qB}$, $t = \dfrac{T}{2} = \dfrac{\pi m}{qB}$ $\left(T = \dfrac{2\pi m}{qB}\right)$

→ $v_{max} = \dfrac{qBR_{max}}{m}$, $KE_{max} = \dfrac{1}{2}mv_{max}^2$.

$$\boxed{\begin{aligned}E_T &= 2NqV\\ &= KE_{max}.\end{aligned}}$$

($V = pd$) $\qquad \therefore N = \dfrac{B^2 q R_{max}^2}{4Vm}$.

28. Moving Coil Galvanometer –

$$\boxed{\begin{aligned}\tau &= MB\sin\theta = MB\\ \tau &= NIAB = C\theta\end{aligned}}$$

($\theta = 90°$)

$1\ ab\text{-}A = 10A$

$\Rightarrow I = \left(\dfrac{C}{NAB}\right)\theta$ $\quad \underline{I \propto \theta}$.

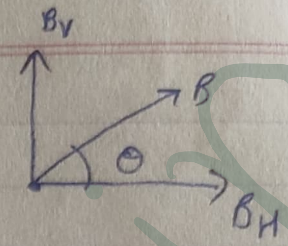
($\tau = -I\alpha$)

($\alpha = -\omega^2\theta$)

Current sensitivity (CS) $= \dfrac{\theta}{I} = \dfrac{NAB}{C}$

Voltage sensitivity (VS) $= \dfrac{\theta}{V} = \dfrac{\theta}{IR} = \dfrac{NAB}{CR}$.

B_H is always from Geographic S to N.



Geo-magnetism-

→ angle of declination $\phi = 11.3^\circ$

→ angle of dip $= \theta$

$$\tan\theta = \frac{B_V}{B_H} = \frac{B\sin\theta}{B\cos\theta}$$

$\left(\begin{array}{l}\theta_{polar} = 90^\circ \\ \theta_{equator} = 0^\circ\end{array}\right)$

→ $\tan\theta = 2\tan\lambda$ (λ = angle of latitude)

→ θ_a = apparent angle of dip

α = angle between given plane and magnetic meridian

$$\tan\theta_a = \frac{\tan\theta}{\cos\alpha}$$

If dip circle is rotated by $90^\circ \Rightarrow$ apparent angle of dip $= \theta'_a$.

$$\cot^2\theta_a + \cot^2\theta'_a = \cot^2\theta.$$

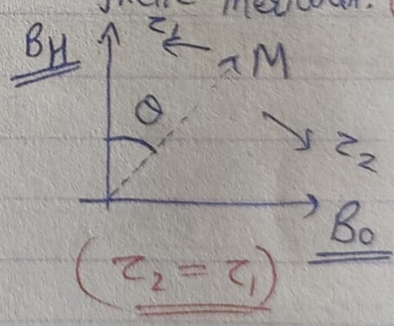
1. **Tangent Galvanometer** (Moving Magnet type) coil is in plane of magnetic meridian. $(B_0 \perp B_H)$

$MB_H\sin\theta = MB_0\sin(90-\theta)$.

→ $B_0 = B_H\tan\theta = \dfrac{\mu_0 NI}{2R}$

$$\underline{I = K\tan\theta} \quad \left(K = \frac{2B_H R}{\mu_0 N}\right)$$

$\left[\begin{array}{l}\text{At } \theta = 45^\circ \\ I = K\end{array}\right]$



2. **Vibration ~~Galvanometer~~ Magnetometer** (Oscillation Magnetometer)

→ $\tau_R = -I\alpha = MB_H\sin\theta$

$= MB_H\theta.$

$\omega^2 = \dfrac{MB_H}{I}$

$$\therefore T = 2\pi\sqrt{\frac{I}{MB_H}}.$$

3. **Neutral Point** - $\underset{\substack{\downarrow \\ \text{due to coil/magnet, etc.}}}{B_{eq}} = B_H.$

Magnetic Materials -

1. $\vec{H} = \dfrac{\vec{B_0}}{\mu_0}$ = magnetising field / magnetisation intensity (unit = H = A/m)

2. $\vec{I} = \dfrac{\vec{M}}{V}$ = intensity of magnetisation (unit = I = A/m)

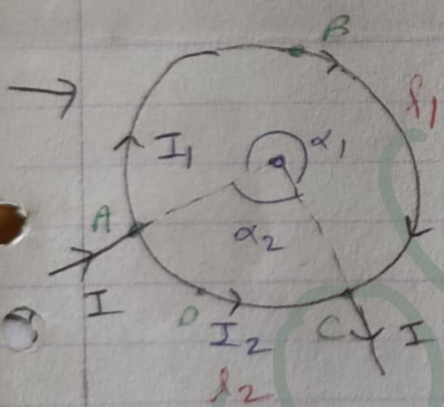
3. $\chi_m = \dfrac{I}{H}$ = susceptibility (more $\chi_m \Rightarrow \uparrow$ ease of magnetisation)

4. $\mu_r = \dfrac{\mu}{\mu_0}$ $\qquad \mu = \dfrac{B_M}{H}$ (unit = Weber = Henry-ampere $= A^2/m$)

5. $\left\{ \begin{aligned} B_M &= B_0 + B_i \\ &= \mu_0 H + \mu_0 I \end{aligned} \right\}$ $\qquad \mu_r = \dfrac{\mu}{\mu_0} = 1 + \chi_m.$

μ_r (vacuum) = 1

μ_r (air) = 1.04



$B_{ABC} = \dfrac{\mu_0}{4\pi} \dfrac{I_1 \alpha_1}{R}$

$B_{ADC} = \dfrac{\mu_0}{4\pi} \dfrac{I_2 \alpha_2}{R}$

$\dfrac{B_{ABC}}{B_{ADC}} = \dfrac{I_1 \alpha_1}{I_2 \alpha_2}$

$\alpha_1 = \dfrac{l_1}{R}$ $\qquad \alpha_2 = \dfrac{l_2}{R}$

$\Rightarrow \dfrac{\alpha_1}{\alpha_2} = \dfrac{l_1}{l_2}.$

$I \propto \dfrac{1}{R}$ (Parallel)

$R = \dfrac{\rho l}{A}$ $\qquad \therefore R \propto l$ $\qquad \Rightarrow I \propto \dfrac{1}{l}$

$\Rightarrow \dfrac{I_1}{I_2} = \dfrac{l_2}{l_1}.$

$$\boxed{\dfrac{B_{ABC}}{B_{ADC}} = \left(\dfrac{I_1}{I_2}\right)\left(\dfrac{\alpha_1}{\alpha_2}\right) = \left(\dfrac{l_2}{l_1}\right)\left(\dfrac{l_1}{l_2}\right) = 1}$$

$\therefore \boxed{B_{centre} = 0}.$