Capacitors:-

→ $Q = CV$ unit of C = Farad (F) = C/V.

→ capacitance of a conductor depends on-

(i) shape and size of the conductor (size ↑ ⇒ C ↑)

(ii) surrounding (environment) medium (K or ε_r ↑ ⇒ C ↑)

(iii) presence of other conductors-(presence of neutral conductors ⇒ C ↑).

→ Sharing of charges-

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{(Q_1 + Q_2)}{C_1+C_2} = \frac{Q}{C_1+C_2}$$

$$Q_1' = \left(\frac{C_1}{C_1+C_2}\right)Q \qquad Q_2' = \left(\frac{C_2}{C_2+C_1}\right)Q$$

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

When two capacitors are connected-

(i) if plates of similar polarity are connected ⇒ take Q_1, Q_2, V_1, V_2 as +ve

(ii) if plates of opp. polarity are connected ⇒ take charge and potential of any one plate as −ve.

→ $U_{cap.} = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

$W_{battery} = QV$ ∴ $\frac{1}{2}QV$ is lost as heat. ($\frac{1}{2}$ of W_b)

→ Spherical capacitor-

outer sphere is earthed ⇒ $C = \frac{4\pi\varepsilon_0 R_2R_1}{R_2 - R_1}$

isolated conducting sphere ⇒ $C = 4\pi\varepsilon_0 R$

inner sphere is earthed ⇒ $C = \frac{4\pi\varepsilon_0 R_2^2}{R_2 - R_1}$

For earth $C = 4\pi\varepsilon_0 R_e = 711\ \mu F$

→ Cylindrical capacitor - $C = \frac{2\pi\varepsilon_0 l}{\ln R_2/R_1}$

→ Parallel plate capacitor - $C = \frac{A\varepsilon_0}{d}$ (A = overlapping area).

$E_{net} = \frac{Q}{2A\varepsilon_0} + \frac{Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

→ **In parallel plate capacitor -**

F on any one plate due to the other $= E_{plate} Q$

(electric pressure - $P = \frac{F}{A} = \frac{Q^2}{2\varepsilon_0 A^2}$)

$= \frac{\sigma}{2\varepsilon_0} Q = \frac{Q^2}{2A\varepsilon_0}$

→ Combination of N-identical drops into one big drop -

$V_{Big} = N^{2/3} V \qquad C_{Big} = N^{1/3} C.$

→ Apply kirchoff's law for circuits (loop law).

→ Series combination - $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$

(same charge)

$V_1 = \left(\frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}\right) V \qquad V_2 = \left(\frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}\right) V$

All capacitor must be initially uncharged to apply the series, parallel combination results.

$U_{combination} = \frac{Q^2}{2C_{eq}}$.

→ Parallel Combination - $C_{eq} = C_1 + C_2 + C_3 + \dots$

$Q_1 = \left(\frac{C_1}{C_1 + C_2 + C_3}\right) Q$ (same pd).

→ When capacitors are not initially uncharged -

$\Delta H = U_{battery} - \Delta U_{capacitor}$

($\Delta U_{capacitor} = U_f - U_i$).

→ Parallel plate capacitors - apply series, parallel.

R-C Circuit -

→ At $t=0$ ⇒ charge on capacitor $= 0$

∴ pd across capacitor $= 0$

∴ Replace capacitor with conducting wire.

→ At $t = \infty$ ⇒ charge on capacitor $=$ Maximum ($C\varepsilon$)

⇒ pd across capacitor $=$ max (ε)

∴ current in branch with capacitor $= 0$

(like open circuit)

$e = 2.7$

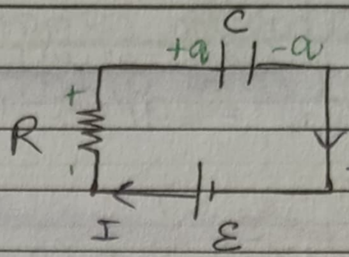
$\frac{1}{e} = 0.37$

$\therefore 1 - \frac{1}{e} = 0.63$

Charging circuit -

Apply loop law to find I

$I = \frac{dq}{dt}$



(initially uncharged capacitor)

→ $q(t) = q_0 (1 - e^{-t/\tau})$

q_0 = charge at $t = \infty$

= max charge on capacitor = $\underline{C\varepsilon}$.

$\tau = R_{eq} C_{eq} = RC$.

→ After one time constant ($t = \tau$)

$q = q_0 \left(1 - \frac{1}{e}\right) = \underline{0.63 q_0}$.

→ pd across capacitor - $V_c = \frac{q}{C} = \frac{q_0}{C}(1 - e^{-t/\tau})$

$\left(\frac{q_0}{C} = V_0 = \text{pd at } t = \infty\right)$

→ $I = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-t/\tau} = I_0 (e^{-t/\tau})$

$\left(I_0 = \frac{\varepsilon}{R} = \text{current at } t = 0\right)$.

→ pd across resistance - $V_R = IR = \varepsilon (e^{-t/\tau})$.

→ $\Delta H = \frac{1}{2} C \varepsilon^2$.

For many resistances (bigger circuit) -

(i) replace battery with internal resistance or conducting wire (if there is no internal resistance).

(ii) Find R_{eq} across capacitor.

(iii) Find $\tau = R_{eq} C_{eq}$.

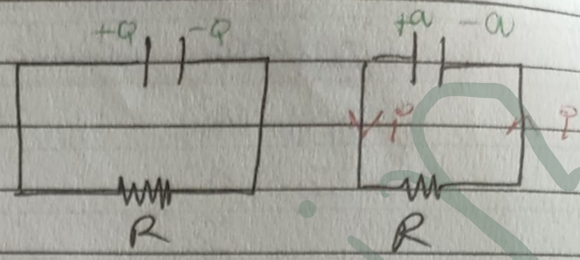
(iv) Find q_0 (max. charge on capacitor at $t = \infty$) using concept of $t = \infty$.

Then $q = q_0 (1 - e^{-t/\tau})$.

For dielectric $\Rightarrow$ $E_{ext} \neq E_{induced}$ $\therefore E_{net} \neq 0$

For conductor $\Rightarrow$ $E_{ext} = E_{induced}$ $\therefore E_{net} = 0$.

Discharging circuit



$\rightarrow$ $q(t) = q_0(e^{-t/\tau})$.

$(q_0 = C\varepsilon)$

$\rightarrow$ $i(t) = I_0(e^{-t/\tau})$ $\left(I_0 = \frac{\varepsilon}{R}\right)$.

Dielectric.

$\rightarrow$ $K = \varepsilon_r = \frac{E_{ext}}{E_{net}}$. $\quad E_{net} = E_{ext} - E_i = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_i}{\varepsilon_0}$.

$\therefore \varepsilon_r = 1$ (for air)

$\varepsilon_r = \infty$ (for conductor $(E_{net} = 0)$).

$\rightarrow$ $E_{net} = \frac{E_{ext}}{k} = \frac{\sigma}{k\varepsilon_0}$ $\quad \therefore E_{medium} = \frac{E_{air}}{k}$.

$\rightarrow$ $\sigma_i = \sigma\left(1 - \frac{1}{k}\right)$ $\quad Q_i = Q\left(1 - \frac{1}{k}\right)$.

$\rightarrow$ $C_{air} = \frac{A\varepsilon_0}{d}$ $\quad C_{medium} = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\sigma A}{\frac{\sigma}{k\varepsilon_0} d}$

$\therefore C_{medium} = \frac{kA\varepsilon_0}{d} = k\,C_{air}$.

$\rightarrow$ Polarization vector $= \vec{P} = \frac{\text{Total dipole moment of dielectric}}{\text{Volume}}$

$\left(n = \frac{N}{V}\right)$. $\vec{p}$ = dipole moment due to each molecule of dielectric $= \frac{N\vec{p}}{V} = n\vec{p}$

$\rightarrow$ $\vec{P} = \frac{q_p \times d}{V} = \frac{(\sigma_i A)d}{V}$ $\quad (A \times d = V)$

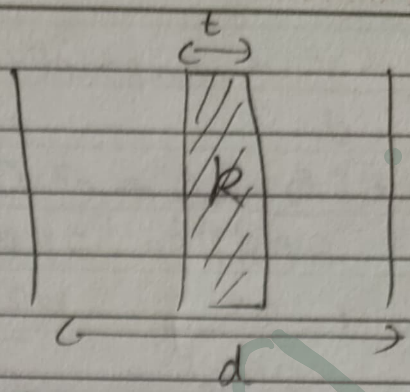
$\therefore \vec{P} = \sigma_i$.

$\rightarrow$ Dielectric strength $= E_{breakdown} = \frac{V_{breakdown}}{d}$.

$\left(V_b = \frac{(Q_{eq})_{max}}{C_{eq}}\right)$

→ **Partially filled dielectric**

$$C = \frac{A\varepsilon_0}{(d-t)+\left(\frac{t}{k}\right)}$$



→ Plate separation division ⇒ series
Plat area division ⇒ parallel.

Force on dielectric

→ when dielectric is completely inserted ⇒ $F_{net} = 0$.
→ dielectric is partially inserted –

(i) at constant V ⇒ $F = \frac{1}{2}V^2\left(\frac{dC_{eq}}{dx}\right)$

(ii) at constant charge ⇒ $F = \frac{Q^2}{2C^2}\left(\frac{dC_{eq}}{dx}\right)$.

$$\frac{dC_{eq}}{dx} = \frac{b\varepsilon_0(k-1)}{d}$$ (d = distance between plates. b = length of plate).

→ Dielectric is inserted (Q = constant) Q, F
(capacitor is not connected with battery).

$C_i = C$ $\left(V_i = \frac{Q}{C} \quad V_f = \frac{Q}{kC}\right)$

$C_f = kC$

$U_i = \frac{Q^2}{2C}$ $U_f = \frac{Q^2}{2kC}$ | $F_i = F_f = \frac{Q\sigma}{2\varepsilon_0}$

$E_i = \frac{\sigma}{\varepsilon_0}$ $E_f = \frac{\sigma}{k\varepsilon_0}$

V, E

→ Dielectric is inserted (V = constant) (battery is connected)

$Q_i = CV$ $Q_f = kCV$

$U_i = \frac{1}{2}CV^2$ $U_f = \frac{1}{2}kCV^2$.

$$E = \frac{V}{d} = \text{constant.}$$

$$F_i = \frac{Q^2}{2A\varepsilon_0} = \frac{C^2V^2}{2A\varepsilon_0}$$

$$F_f = \frac{Q^2}{2A\varepsilon_0} = \frac{K^2C^2V^2}{2A\varepsilon_0} = \underline{\underline{K^2 F_i}}.$$

NCERT Points-

1. In case of dielectrics, we consider the situation where the induced dipole moment is in the direction of the external field and is proportion to the field strength. Substances for which this assumption is true are called linear isotropic dielectrics.

2. The dipole moment per unit volume is called polarisation and is denoted by P.

   $P = \chi_e E$ χ_e = electric susceptibility of the dielectric medium (constant characteristic of the dielectric).

3. A capacitor is a system of two conductors separated by an insulator.

4. Q is called the charge of the capacitor, though it is the charge on one of the conductors — the total charge of the capacitor is zero.

5. Capacitor of fixed capacitance
   Capacitor of varying capacitance

6. The maximum electric field that a dielectric medium can withstand without break-down (of its insulating property) is called its dielectric strength. For a capacitor to store large amount of charge without leaking, its capacitance should be high enough

so that the potential difference and hence, the electric field do not exceed the break-down limits.

7. Van de Graaf generator can build up high voltages of the order of a few million volts. The resulting large electric fields are used to accelerate charged particles (electrons, protons, ions) to high energies needed for experiments to probe the small scale structure of matter.

8. Breakdown field of air $= 3 \times 10^6$ V/m.