Vectors

1. Vector has both direction and magnitude.

2. <u>Types of vectors</u>-

① Parallel vectors - same direction
② Antiparallel vectors - opposite direction
③ Equal vectors - same magnitude, same direction
④ Negative vectors - same magnitude, opposite direction
⑤ Co-initial vectors - vectors having same initial point
⑥ Colinear vectors - vectors which lie along the same straight line
⑦ Coplanar vectors - vectors in the same plane.
⑧ Concurrent vectors - vectors which pass through a common point
⑨ Polar vectors - vectors having starting point or point of application Eg - displacement, force.
⑩ Axial vectors - used to define angular quantities. Axial vector is along the direction of the axis of rotation decided by the right hand thumb rule. Eg - torque, angular acceleration.

3. <u>Unit vector</u> (magnitude = 1). $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

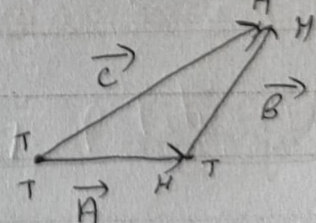
4. When $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$|\vec{A}| = \sqrt{A_1^2 + A_2^2 + A_3^2}$.

5. <u>Null / zero Vector</u> - magnitude = 0 $\vec{O}$.

6. <u>Triangle law of vector addition</u>

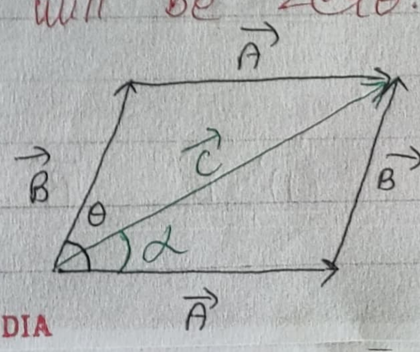
(T = tail) (H = head).



$\vec{A} + \vec{B} = \vec{C}$.

- If the vectors form a closed loop in the same sense, then sum of all vectors will be zero.

7. <u>Parallelogram Law of Vector Addition</u>



$\vec{C} = \vec{A} + \vec{B}$

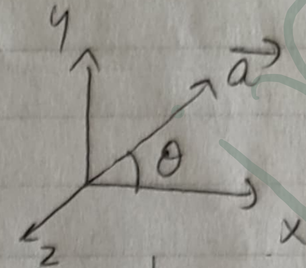
$|\vec{C}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$.

→ $\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$ (α = angle between resultant vector and $\vec{A}$.

8. If $\vec{C} = \vec{A} - \vec{B}$ ⇒ $|\vec{C}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$.

9. When $\vec{a} = x\hat{i} + y\hat{j}$ $\quad x = a\cos\theta$

$= a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$. $\quad y = a\sin\theta$



→ $a_x = a\cos\alpha$

$a_y = a\cos\beta$

$a_z = a\cos\gamma$

$\cos\alpha, \cos\beta, \cos\gamma$ = direction cosines.

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$.

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

10. <u>Scalar/Dot Product.</u>

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$ (θ = angle between $\vec{A}$ and $\vec{B}$)

→ if $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

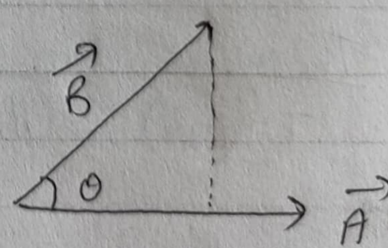
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

If two vectors are ⊥ to each other, their dot product will be zero.

11. <u>Projection of $\vec{B}$ on $\vec{A}$</u>



→ <u>Scalar form</u> - $\dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|} = \hat{A}\cdot\vec{B}$.

→ <u>Vector form</u> - $\left(\dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|}\right)\hat{A} = (\hat{A}\cdot\vec{B})\hat{A}$.

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.

Vector / Cross Product

→ $\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \, \hat{n}$ ($\hat{n}$ is a vector $\perp$ to $\vec{A}$ and $\vec{B}$, decided by the right hand thumb rule).

→ $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$(\vec{A} \times \vec{B}) = -(\vec{B} \times \vec{A})$.

→ $\hat{n} = \pm \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$\hat{i} \times \hat{j} = \hat{k}$ $\quad$ $\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ $\quad$ $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{k} \times \hat{j} = -\hat{i}$ $\quad$ $\hat{i} \times \hat{k} = -\hat{j}$

→ $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_y B_z - B_y A_z) - \hat{j}(A_x B_z - B_x A_z) + \hat{k}(A_x B_y - B_x A_y).$$

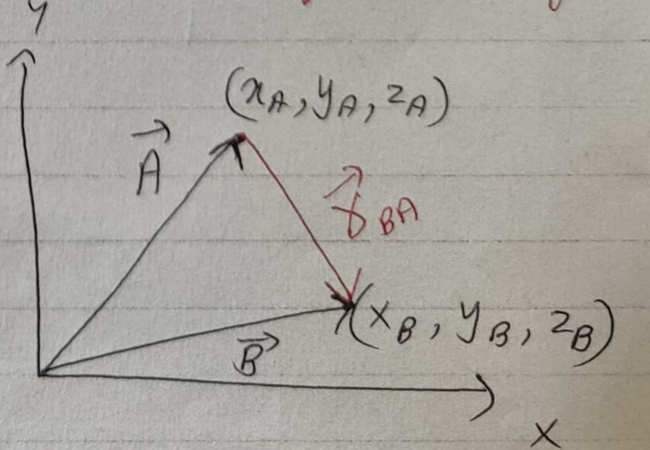
13. If $\vec{A}$ and $\vec{B}$ are two adjacent sides of a parallelogram,

area of parallelogram $= |\vec{A} \times \vec{B}|$.

→ Area of triangle $= \frac{1}{2} |\vec{A} \times \vec{B}|$

→ Area of parallelogram $= \frac{1}{2} |\vec{d_1} \times \vec{d_2}|$ (d_1, d_2 = diagonals of parallelogram).

14. Position Vector



$\vec{A}$ = position vector of A

$= x_A \hat{i} + y_A \hat{j} + z_A \hat{k}$.

$\vec{B}$ = position vector of B

$= x_B \hat{i} + y_B \hat{j} + z_B \hat{k}$.

→ $\vec{r}_{BA}$ = position vector of $\vec{B}$ with respect to A

$= \vec{B} - \vec{A}$

$= (x_B - x_A)\hat{i} + (y_B - y_A)\hat{j} + (z_B - z_A)\hat{k}$

→ A vector can be split into infinite components.

→ If $\vec{A}$ and $\vec{B}$ are two vectors whose resultant vector makes an angle α with $\vec{A}$ and β with $\vec{B}$, then –

if $|\vec{A}| < |\vec{B}| \Rightarrow \alpha > \beta$

if $|\vec{A}| > |\vec{B}| \Rightarrow \alpha < \beta$

if $|\vec{A}| = |\vec{B}| \Rightarrow \alpha = \beta$.

→ In a triangle, the sum of two sides is always greater than the third side.

→ 4 minimum no. of vectors in different planes can be added to give zero resultant.

→ No. of rectangular components into which a vector can be split – (i) in its own plane = 2

(ii) in space = 3.