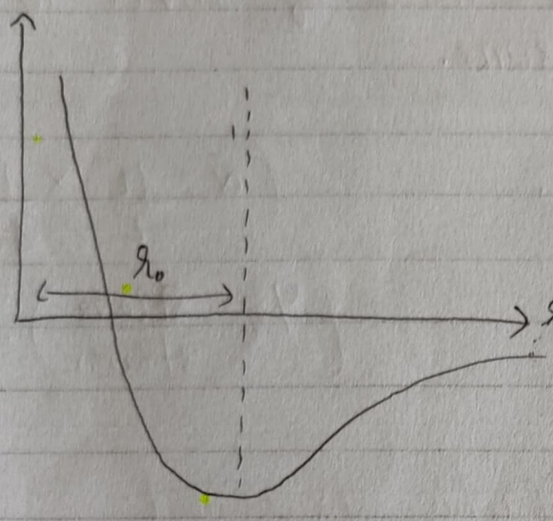
Temperature And Thermal Expansion

1. Zeroth Law of Thermodynamics - two objects (or systems) are said to be in thermal equilibrium if their temperatures are same.

2. $$\frac{K-273}{100} = \frac{C}{100} = \frac{F-32}{180} = \frac{X - MP_\omega}{BP_\omega - MP_\omega}$$

$$\frac{\Delta K}{100} = \frac{\Delta C}{100} = \frac{\Delta F}{180} = \frac{\Delta X}{BP_\omega - MP_\omega}$$

3.



- assymetry in PE curve about equilibrium position r_0 is responsible for thermal expansion.

- Solids can expand in one dimension (linear expansion), 2D (superficial expansion), 3D (volumetric expansion).
- Liquids and gases usually suffer change in volume only.

4. $l = l_0(1 + \alpha \Delta\theta) = l_0 + \Delta l$ $\quad \therefore \Delta l = l_0 \alpha \Delta\theta$.

$A = A_0(1 + \beta \Delta\theta)$ $\quad A_0 = l_0^2$ $\quad A = l^2$

$\therefore 2\alpha = \beta$.

$V = V_0(1 + \gamma \Delta\theta)$ $\quad V = l^3$ $\quad V_0 = l_0^3$ $\quad \therefore 3\alpha = \gamma$.

$\therefore \alpha : \beta : \gamma = 1 : 2 : 3$.

5. Contraction on Heating

→ Some rubber like substances contract on heating because transverse vibrations of atoms of substances dominates over longitudinal vibration which is responsible for expansion.

① Bi-metallic strip - made of two strips of substances with different α. used in thermostat to break or make electrical contact. On heating, strip will bend with metal of greater α on outer side. $\alpha_{brass} > \alpha_{steel}$.

② Effect of Temperature on Time period (T) of a simple pendulum

→ $\frac{\Delta T}{T} = \frac{1}{2}\alpha \Delta\theta$ $\left(\because T \propto \sqrt{l} \quad \therefore \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}\right)$

→ Loss of time in a time period $= \Delta T = \frac{1}{2}\alpha \cdot \Delta\theta \cdot T$.

→ ∵ coefficient of linear expansion (α) of Invar is very small, ∴ pendulums are made of Invar to show the correct time in all seasons.

③ Thermal strain $= \frac{\Delta L}{L} = \alpha \Delta\theta$. $\left(\alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta\theta}\right)$

Thermal stress $= Y \alpha \Delta\theta$ $\left(Y = \frac{\text{stress}}{\text{strain}}\right)$.

④ Expansion of Cavity

→ may be imagined as photographic enlargement.

6. Thermal Expansion In Liquids

→ Actual ↑ in V_{liquid} = apparent ↑ in V_{liquid} + ↑ in $V_{container}$.

→ γ_a (due to ↑ in V_{liquid} if expansion of vessel is not taken into account).

→ γ_r (due to actual increase in V_{liquid}.)

$\gamma_{real} = \gamma_{apparent} + \gamma_{vessel}$.
(γ_r) (γ_a) (γ_v)

→ Apparent change in Volume of liquid relative to vessel

$\Delta V_{app} = V(\gamma_{real} - \gamma_{vessel})\Delta\theta = V(\gamma_r - 3\alpha)\Delta\theta$

(α = coefficient of linear expansion of vessel)

If $\gamma_r > \gamma_v \Rightarrow$ level of liquid will rise
$\gamma_r < \gamma_v \Rightarrow$ level of liquid will fall
$\gamma_r = \gamma_v \Rightarrow$ level of liquid will remain same.

Heat

1 BTU = 252 calorie (BTU = British Thermal Unit).
1 J = 4·2 cal
1 J = 10^7 erg.

2. Specific heat (S or c). - $c = \frac{1}{m}\frac{dQ}{dT}$.

$\boxed{Q = ms\Delta\theta}$ c_{water} = 1 cal/g °c
= 4200 J/kg-K.

3. Molar heat Capacity (C)

$C = c \times$ mol. wt. $= \frac{1}{m}\frac{dQ}{dt} \times M$ $\left(\mu = \text{no. of moles} = \frac{m}{M}\right)$

$\therefore \boxed{C = \frac{1}{\mu}\frac{dQ}{dT}}$

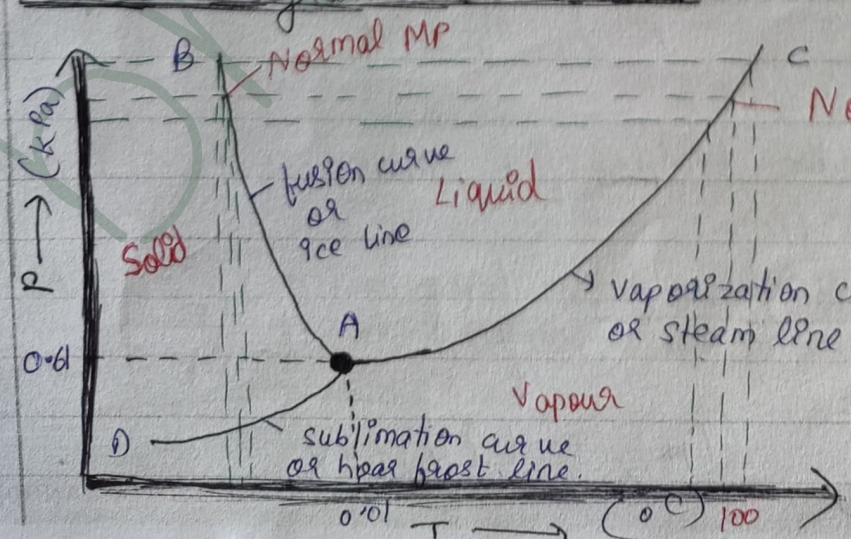
4. Thermal Capacity / Heat Capacity = mass x specific heat
= ms.

5. Latent Heat $\boxed{Q = mL}$ For ice/water
L_f = 80 cal/g
L_v = 540 cal/g.

6. Hoar Frost
Direct conversion of vapour to solid. Reverse of sublimation.

7. Regelation - the melting of ice caused by pressure and its resolidification when the pressure is removed.

8. Phase Diagram For Water



A = triple point
where all three phases are in equilibrium.

∴ If $P \uparrow \Rightarrow MP \downarrow$, $BP \uparrow$
$P \downarrow \Rightarrow MP \uparrow$, $BP \downarrow$.

9. Principle of Calorimetry ⇒ Heat lost by one body = Heat gained by other body.

Modes of Heat Transfer

10. Conduction - the process in which the material takes active part by molecular action and energy is passed from one particle to another. (predominant in solids). - due to free e^- / vibration of molecules.
Convection - the transfer of energy by actual motion of particles of the medium from one place to another is called convections. (predominant in fluids - liquids & gases). - actual motion of particles.
Radiation (Fastest) - quickest mode of transmission of heat. (No medium required) - electromagnetic radiation.

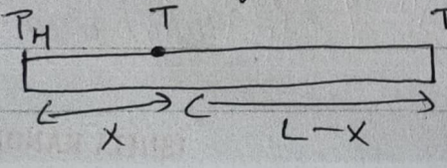
11. Thermal Conduction - $\boxed{\frac{Q}{t} = -KA\frac{dT}{dx}}$
K = thermal conductivity
A = ⊥ to heat flow.

Q = heat transferred
t = time interval
A = area of x-section
dT = diff. in Temp.
dx = lateral distance.

$$\boxed{\frac{Q}{t} = \text{heat current} = i_H = \frac{KA(T_H - T_C)}{L} = \frac{\Delta T}{L/KA} = \frac{\Delta T}{R_H}}$$

$\boxed{R_H = \frac{L}{KA}}$ (like $I = \frac{V}{R}$) L is along the direction of flow of heat.

12. Steady State - If the temp. of a cross-section of slab remains constant with time.
For a conductor in steady state, there is no absorption or emission of heat at any cross-section.



$(i_H)_X = (i_H)_{L-X}$.

13. Thermal Conductivity (K) - depends on nature of material.

order - $Ag > Cu > Au > Al$

K < max = Ag, min. = Freon.

14. Equivalent Conductivity

I. For Slabs in Series (same i_H).

$$R_{eq} = R_1 + R_2$$

$$\frac{L_1 + L_2}{K_{eq} A} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}$$

II. For Slabs in Parallel (same ΔT)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K_{eq} A_1}{L} = \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L}$$

15. Growth of Ice On Lakes -

Time taken by ice to grow from thickness x_1 to x_2 is -

$$t = t_2 - t_1 = \frac{1}{2} \frac{\rho L}{K T} (x_2^2 - x_1^2)$$

16. Thermal Radiation

→ Radiation intensity is measured with a special device known as bolometer.

17. Absorptive Power / Absorptive Coefficient (a) - $a = \frac{Q_a}{Q_i}$

Q_a = radiation absorbed

Q_i = incident radiation

18. Spectral Absorptive Power - $a_\lambda = \frac{Q_{a_\lambda}}{Q_\lambda}$

→ At a given wavelength - $\boxed{a = \int_0^\infty a_\lambda \, d\lambda}$

For ideal black body, $a = a_\lambda = 1$.

19. Emissive power (E) = amount of heat radiation emitted by unit surface area in unit second.
Spectral Emissive power (E_λ) = E at a given wavelength.

$$\boxed{E = \int_0^\infty E_\lambda \, d\lambda}$$

20. Emissivity (e)

→ Absolute emissivity / emissivity - Radiation given out by unit surface area of a body in unit time corresponding to unit temperature diff. w.r.t surroundings.

→ Relative emissivity (e_r)

$$\boxed{e_r = \frac{Q_{GB}}{Q_{IBB}} = \frac{\text{Radiation emitted by gray body}}{\text{Radiation emitted by ideal black body}}}$$

$\boxed{\text{For IBB} \rightarrow e_r = 1}$ $\quad (0 < e_r < 1)$.

21. $\boxed{Q = Q_r + Q_a + Q_t}$

$\left(r = \frac{Q_r}{Q}\right)\left(a = \frac{Q_a}{Q}\right)\left(t = \frac{Q_t}{Q}\right)$

Q = total incident radiation
a = absorbed
t = transmitted
r = reflected.

$\boxed{r + a + t = 1}$

$r = 1, a = 0, t = 0 \Rightarrow$ Perfect reflector

IBB $\Leftarrow r = 0, a = 1, t = 0 \Rightarrow$ Ideal absorber

diathermanous $\Leftarrow t = 1, a = 0, r = 0 \Rightarrow$ Perfect transmitter

22. IBB $\Rightarrow a = a_\lambda = 1$. $\quad r = t = 0 \quad e_r = 1$.

At low temp $\Rightarrow$ perfect absorber
at high temp. $\Rightarrow$ perfect emitter.

Kirchoff's Law $\boxed{\frac{e_\lambda}{a_\lambda} = E_\lambda = \text{constant}}$ $\therefore$ $\boxed{e_\lambda \propto a_\lambda}$

Good absorbers are good emitters. Bad absorbers are bad emitters.

24. Fraunhoffer's Lines - They are dark lines in the spectrum of the Sun. At the time of total solar eclipse, direct light rays emitted from photosphere (inside; 10^7 K) cannot reach on earth and only rays from chromosphere (outside; 6000K) are able to reach on the Earth surface. At this time we observe bright Fraunhoffer lines. When white light emitted from the central core of the sun - photosphere passes through its atmosphere - chromosphere, some of the radiations are absorbed by the gases present, resulting in dark lines.

25. Stefan's Law - the amount of radiation emitted per second per unit area by IBB $\propto$ fourth power of its absolute temperature.

$\boxed{E \propto T^4}$ T = temp. of IBB in K.

$\boxed{E = \sigma T^4}$ σ = Stefan's constant = 5.67×10^{-8} W/m^{2}K^4 (universal constant).

$Q_{IBB} = \sigma A T^4 t$ $\qquad$ $Q_{GB} = e_r A \sigma T^4 t$

(A = area, t = time).

26. Rate of Emission of Radiation

Temp. of surrounding = T_0. Temp. of black body = T

($T_0 < T$).

$E_{IBB} = \sigma T^4$ $\qquad$ $E_{surrounding} = \sigma T_0^4$

$E = E_{IBB} - E_{surr.} = \sigma (T^4 - T_0^4)$.

$Q_{IBB} = \sigma A (T^4 - T_0^4) t$. $\qquad$ $Q_{GB} = e_r \sigma A (T_4^4 - T_0^4) t$.

(net radiation loss)

→ Rate of loss of heat (IBB) - $\boxed{R_H = \frac{dQ}{dt} = \sigma A (T^4 - T_0^4)}$

→ Rate of fall in temp. $\boxed{R_F = \frac{dT}{dt} = \frac{\sigma A}{ms}(T^4 - T_0^4)}$

$\left(\frac{dQ}{dt} = ms\frac{dT}{dt}\right)$ ∴ $R_H \propto A$

$R_F \propto \frac{A}{ms} \propto \frac{A}{\rho V s}$

→

	Body	R_H	R_F
①	Sphere (solid) (same ρ, s, T, T_0)	$R_H \propto r^2$	$R_F \propto \frac{1}{r}$
②	solid sphere (different ρ, s)	$R_H \propto r^2$	$R_F \propto \frac{1}{r\rho s}$
③	Different shape	$R_H \propto A$	$R_F \propto \frac{A}{ms} \propto \frac{A}{\rho V s}$

27. Newton's Law of Cooling - Rate of cooling $\left(\frac{dT}{dt}\right)$ is directly proportional to excess of temp. of the body over that of the surrounding when $(T - T_0) \not> 35^\circ C$

$\boxed{-\frac{dT}{dt} \propto (T - T_0)}$

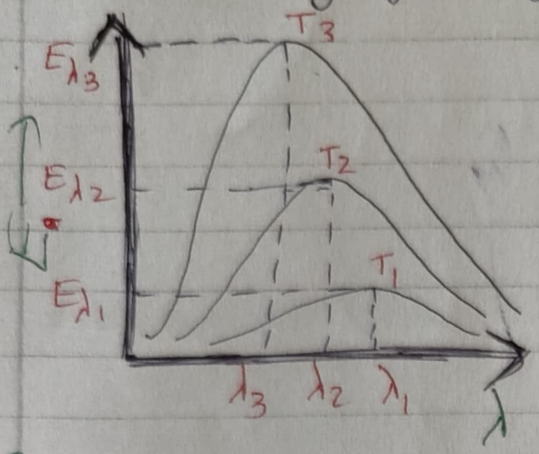
$\boxed{-\frac{dT}{dt} = K(T - T_0)}$

→ If temp. of body decreases from T_1 to T_2 and surrounding temp. = T_0 ,

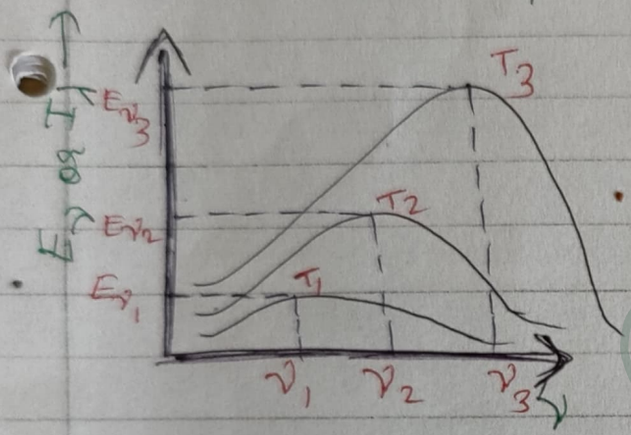
avg. excess temp. = $\boxed{\frac{T_1 + T_2}{2} - T_0}$

$\boxed{\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]}$

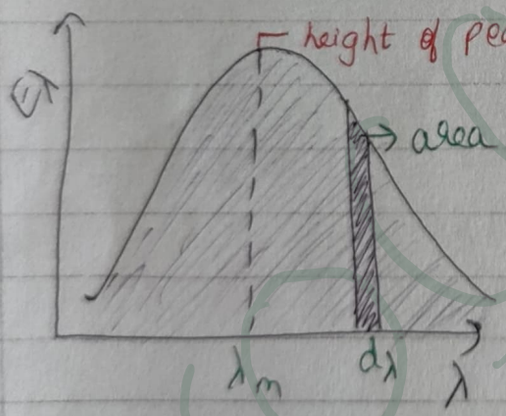
28. Spectral Energy distribution curve of Black Body Radiation
practically given by – Lumeas & Pringshom
Mathematically given by – Plank.



$T_3 > T_2 > T_1$
$\lambda_3 < \lambda_2 < \lambda_1$



$T_3 > T_2 > T_1$
$\nu_3 > \nu_2 > \nu_1$



$\lambda_m \propto \frac{1}{T}$

$\lambda_m T = b$

$E_{\lambda_m} \propto T^5$

Area $\int_0^\infty E_\lambda \, d\lambda = E = \sigma T^4$ $\quad \therefore \frac{A_1}{A_2} = \left[\frac{T_1}{T_2}\right]^4$

b = Wein's constant = 2.89×10^{-3} mk.

29. Solar Constant (S) – S = 1340 W/m² = 1.937 Cal/cm²-minute.

$S = \frac{\sigma T^4 R^2}{d^2}$

R = radius of sun
d = Radius of earth's orbit around the sun.
T = temp. of sun.

T = 5732 K

KTG

$(1\ lit. = 10^{-3} m^3 = 10^3 cm^3 = 10^3 ml)$

1. Ideal Gas Equation

$PV = nRT$. $\quad PV = NkT$ $\quad \left(k = \frac{R}{N_A}\right)$

$\frac{P}{\rho} = \frac{RT}{M_w} = \frac{kT}{m}$.

2. Boyle's Law - $P \propto \frac{1}{V}$ $(T = \text{constant})$

Charle's Law - $V \propto T$ $(P = \text{constant})$

Gay Lussac's Law - $P \propto T$ $(V = \text{constant})$

Avogadro's Law - At the same temperature and pressure, equal volumes of all gases contain equal number of molecules

Dalton's Law of Partial Pressure - $P = P_1 + P_2 + \dots$

3. Assumptions of KTG

1. molecules move randomly in all directions
2. all collisions are elastic
3. No attractive or repulsive forces act on the gas molecules $\therefore PE = 0$.
4. Gravitational attraction is ineffective due to extremely small mass.
5. Pressure is exerted by the gas molecules on the walls of the container due to their collisions with the walls.
6. Mean momentum $= 0$ $\quad$ Mean velocity $= 0$ $(\langle v \rangle = 0)$ $\quad (\langle v^2 \rangle \neq 0)$ $\quad (\langle v^3 \rangle = \langle v^5 \rangle = 0)$
7. mean free path $= \lambda$.

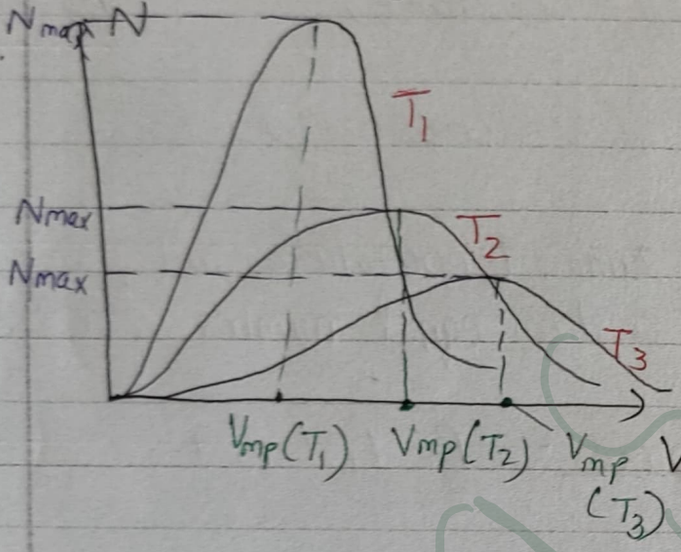
4. $V_{rms} = \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3P}{\rho}}$.

$V_{avg} = \sqrt{\frac{8}{\pi}\frac{P}{\rho}} = \sqrt{\frac{8}{\pi}\frac{RT}{M_w}} = \sqrt{\frac{8}{\pi}\frac{kT}{m}}$.

$V_{mp} = \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2P}{\rho}}$.

$V_{rms} > V_{avg} > V_{mp} > V_{sound}$.

5.



$T_3 > T_2 > T_1$

Maxwell's Velocity Distribution Curve.

6. Due to random motion of molecules - $V_x = V_y = V_z$

$\therefore \boxed{V = \sqrt{3}\,V_x = \sqrt{3}\,V_y = \sqrt{3}\,V_z}$

$\rightarrow \Delta p = 2mV_x \qquad \Delta t = \frac{2l}{V_x}$

$F = \frac{\Delta p}{\Delta t} = \frac{mV_x^2}{l} = \frac{mv^2}{3l}$

$P = \frac{F}{A} = \frac{1}{3}\frac{mv^2}{l \times A} = \frac{1}{3}\frac{mv^2}{vol.}$

$\therefore \boxed{P = \frac{1}{3}\rho V_{rms}^2}$.

$\rightarrow$ Total translational KE per mole $= \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}PV = \frac{3}{2}RT$.

→ Total translational KE per molecule $= \frac{3}{2} kT$.

7. Degrees of freedom -

	Atomicity	Translational	Rotational	Total
①	Monoatomic	3	0	3
②	Diatomic	3	2	5
③	Triatomic linear	3	2	5
④	Triatom (non-linear) / Poly-atomic	3	3	6

At high temp. (> 5000 K), 2 vibrational dof are added.

→ For one molecule of gas
energy associated with each dof $= \frac{1}{2} kT$

∴ total energy $= \frac{f}{2} KT$.

→ $C_V = \frac{fR}{2}$

$C_p - C_V = R$

$C_p = \left(1 + \frac{f}{2}\right) R$

$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$

8. Mean free path of the molecule $\boxed{\lambda_m = \frac{1}{\sqrt{2}\pi d^2 n}}$

$n = \frac{N}{V}$ = no. of molecules per unit volume

d = diameter of the molecule.

Thermodynamics

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1. $U = KE + PE$ ($\to 0$ for ideal gas) — U depends only on temperature.

$\therefore \boxed{U = \frac{nfRT}{2} = nC_VT}$ $\quad \therefore \boxed{\Delta U = nC_V\Delta T}$

2. Sign convention

(a)	Heat gained by the system	+ve
(b)	Heat lost by the system	−ve
(c)	Work done by a system	+ve
(d)	Work done on a system	−ve
(e)	↑ in internal energy	+ve
(f)	↓ in internal energy	−ve

expansion → Work = +ve

compression → W = −ve.

3. Work done by the system ↓ $\boxed{W = \int_{V_i}^{V_f} dW = \int_{V_i}^{V_f} PdV}$

obtained from area under P-V graph.

4. First Law of Thermodynamics (FLOT) - $\boxed{\Delta Q = \Delta U + W}$

5. Different Processes

① Isometric / Isochoric ($\Delta V = 0$) $\therefore W = 0$

$\therefore \Delta Q = \Delta U = nC_V\Delta T$.

Eg - Heating process in a pressure cooker.
- melting of a substance (nearly isochoric)
- when heat is provided to a gas enclosed in a cylinder with rigid walls and a fixed piston.

② Isobaric (P = constant) $\therefore W = P\Delta V = nR\Delta T$.

$\Delta Q = \Delta U + W$

$= nC_V\Delta T + nR\Delta T$

$= n\Delta T(C_V + R) = nC_P\Delta T$.

$\therefore \boxed{\Delta Q = nC_P\Delta T}$.

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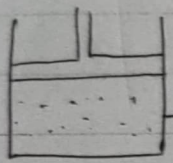
Eg- heating of water at atmospheric pressure.
- melting of solids and boiling of liquids at atmospheric pressure.

③ Isothermal ($\Delta T = 0$) $\therefore \Delta U = 0$. $\Delta Q = W$.

$$W = nRT \ln\left[\frac{V_2}{V_1}\right] = nRT \ln\left[\frac{P_1}{P_2}\right].$$

→ slope of isothermal P-V curve $\left[\frac{dP}{dV} = -\frac{P}{V}\right]$

→  → conducting, slow process

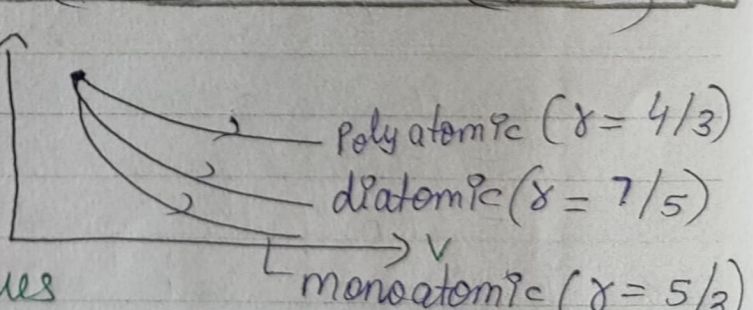
Eg- boiling
- if sudden changes are executed in a vessel of infinite conductivity.

④ Adiabatic ($\Delta Q = 0$) $\therefore \Delta U = -W$.

PV^{γ} = constant.

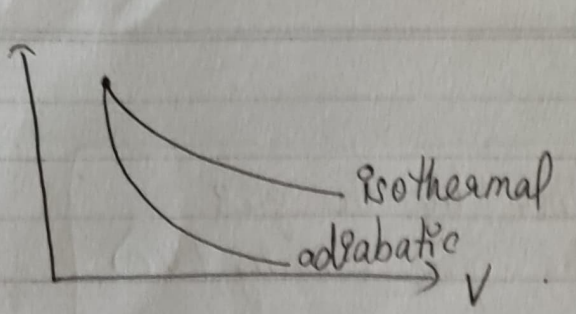
→ slope of adiabatic curve $\left[\frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)\right]$

$$W = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)} = \frac{nR(T_1 - T_2)}{(\gamma - 1)}$$

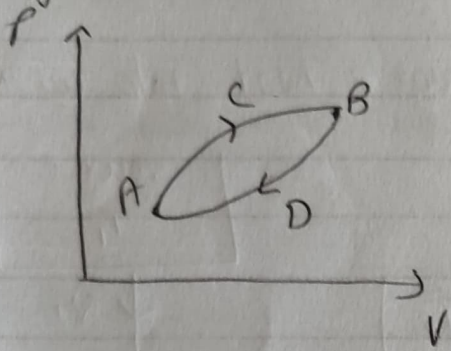


Eg- bursting of a cycle tube
- propagation of sound waves in gaseous medium.
- Gas enclosed in a thermally insulated cylinder with a non-conducting piston. If the gas is compressed/expanded suddenly (fast-process).
- In diesel engines, burning of diesel without spark plug.

→

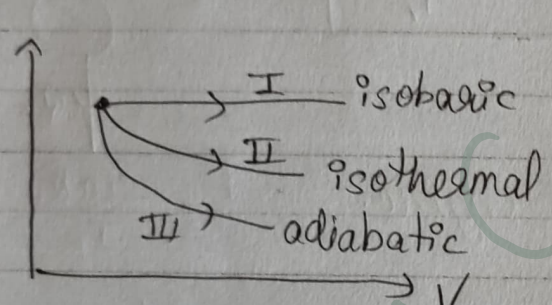


⑤ Cyclic Process $(\Delta T = 0) \quad \therefore \Delta U = 0 \quad \therefore \Delta Q = 0$



$WD = +ve$ (CW)
$WD = -ve$ (ACW)

→



$W_I > W_{II} > W_{III}$.

⑥ Free expansion — isothermal / adiabatic

$\Delta T = 0$ $\quad \Delta Q = 0$
$P = 0$ $\quad W = 0$.

⑦ ~~Polytropic process~~ $PV^n = \text{Constant}$.

→ $\boxed{C = \frac{R}{\gamma - 1} + \frac{R}{1 - n}}$

6. $\boxed{C_v = \frac{R}{\gamma - 1}}$ $\quad \boxed{C_p = \frac{R\gamma}{\gamma - 1}}$ $\quad \boxed{\gamma = 1 + \frac{2}{f}}$

7. Mayer's formula - $\boxed{C_p - C_v = R}$

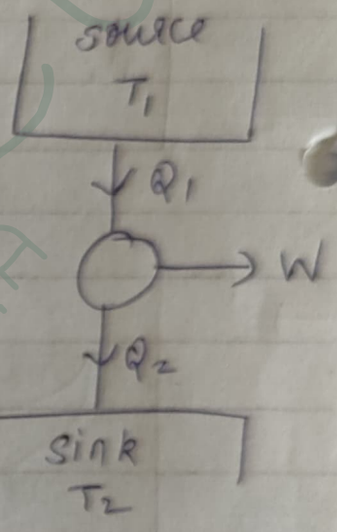
8. **Efficiency of a Cycle (η)**

η = total mechanical work done by the gas / Heat absorbed by the gas (only +ve)
= area of P-V cycle / heat injected into the system.

9. **Heat Engine** → a device which converts heat into work.

$Q_1 = W + Q_2$

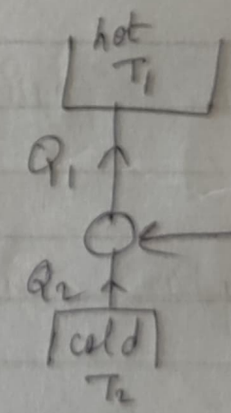
$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$



10. **Refrigerator** - Inverse of heat engine.

$Q_1 = Q_2 + W$

β = Heat extracted from cold reservoir / Work done on refrigerator
$= \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$
= COP



For carnot reversible refrigerator $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$.

$\beta = \frac{T_2}{T_1 - T_2}$.

$\eta = \frac{W}{Q_1}$ → efficiency of heat engine

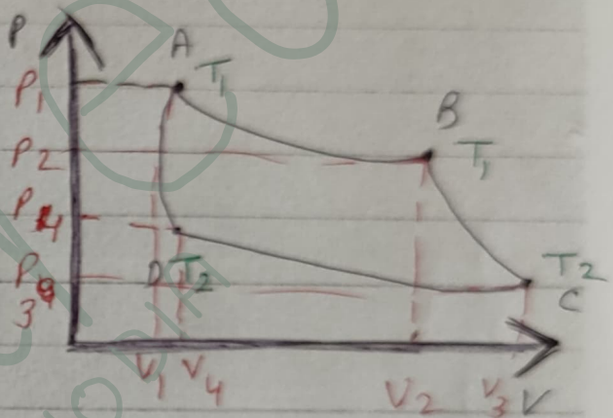
$\beta = \frac{1}{\eta} - 1$.

11. SLOT - the entropy of the Universe never decreases. It increases or remains the same.
- No process is possible whose sole result is the transfer of heat from a colder to a hotter object.

12. Reversible process - $\Delta S = 0$
Irreversible process $\Delta S \geq 0$

13. Entropy. $\boxed{dS = \frac{dQ}{T}}$ $\boxed{\Delta S = \int \frac{dQ}{T}}$

14. Carnot Cycle - Ideal engine
(i) Isothermal expansion A → B
(ii) Adiabatic expansion B → C
(iii) Isothermal compression C → D
(iv) Adiabatic compression D → A.



Isothermal Expansion (A → B) $\Delta U = 0$

$W_1 = Q_1 = nRT_1 \ln \frac{V_2}{V_1}$ (+ve)

$W_2 = \frac{nR(T_1 - T_2)}{\gamma - 1}$ (+ve)

$W_3 = Q_3 = nRT_2 \ln \frac{V_4}{V_3}$ (-ve)

$W_4 = \frac{nR(T_2 - T_1)}{\gamma - 1}$ (-ve).

$\boxed{\frac{V_2}{V_1} = \frac{V_3}{V_4}}$ $\boxed{\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}}$

Carnot Theorem - No irreversible engine (I) can have efficiency greater than carnot reversible engine (R).

$\boxed{\eta_R > \eta_I} \Rightarrow \boxed{1 - \frac{T_2}{T_1} > 1 - \frac{Q_2}{Q_1}}$