INELASTIC = PLASTIC

PROPERTIES OF MATTER (ELASTICITY)

1. **Elasticity** - property of matter by virtue of which body tends to regain its shape and size after the deforming forces are removed.
2. **Rigid body** - a body in which separation between constituent particles does not change $\Rightarrow$ no deformation. (No body is perfectly rigid)
3. **Inelastic or plastic body** W.D in deformation = completely dissipated during deformation (no tendency to regain original shape).
4. **Stress** -

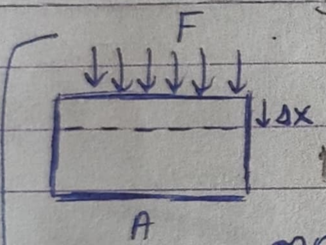
$$\text{Stress}, \sigma = \frac{\text{(Deformation) Force}}{\text{Cross-sectional area} \perp \text{to the force}}$$

normal

$$\left[\sigma = \frac{F}{A}\right]$$

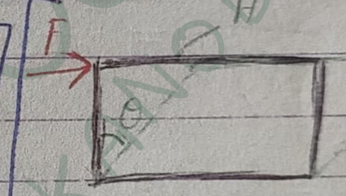
Bulk stress = Pressure

5. **Stress**
   - Normal stress (n)
   - shear stress (s)



$F \perp A$. $\sigma_n = \frac{F}{A}$.

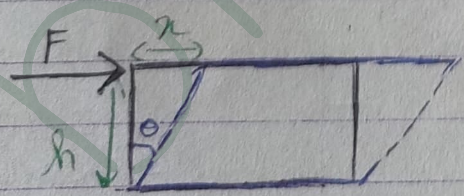
normal stress (longitudinal)



$\sigma_s = \frac{F}{A}$.

shear stress (tangential)

6. **Strain** (ϵ) = $\frac{\text{change in configuration/dimensions}}{\text{original configuration}} = \frac{\Delta D}{D}$.
   - Longitudinal strain $= \Delta L / L = \varepsilon_L$
   - volume (bulk) strain $= \Delta V / V = \varepsilon_V$
   - shear strain = tangential deformation $= \varepsilon_t$



$$\text{shear strain} = \theta = \tan^{-1}\left(\frac{x}{h}\right).$$

(For small angle $\Rightarrow \theta = \frac{x}{h}$.)

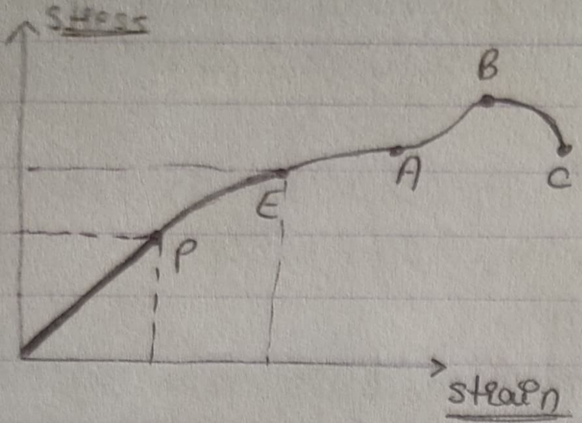
At equilibrium,
$f_{internal} = f_{external}$.

$$\delta = \frac{4 W l^3}{4 b d^3 Y}.$$

METALS ARE MORE ELASTIC THAN RUBBER (STIFF)

★ Experimental ⇒ 0.2 to 0.4
★ Theoritical ⇒ −1 to 0.5

Hooke's Law - stress ∝ strain. ★



Till P ⇒ Hooke's law
P to E ⇒ Elasticity limit (till here, body tends to regains original size)
After E ⇒ body breaks its elasticity limit ⇒ permanent deformation
After A ⇒ keeps on elongating
B = breaking point

★ $\boxed{E = \frac{\text{stress}}{\text{strain}} = \text{modulus of elasticity} = \text{constant}}$

Young's Modulus (Y)
$Y = \frac{FL}{A\Delta L}$

Bulk Modulus (B) or K

Shear modulus (η) modulus of rigidity

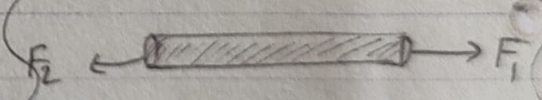
→ $B = \frac{\Delta P}{\Delta V/V} = \frac{-P_{excess}}{\Delta V/V}$ (B is always +ve)

→ $\eta = \frac{F}{A\theta} = \frac{F}{A}\left(\frac{1}{\tan^{-1}(x/h)}\right)$

8. Compressibility - C

$\boxed{C = \frac{1}{B}}$

Trick



$\Delta L = \left(\frac{F_1+F_2}{2}\right)\left(\frac{L}{AY}\right)$

9. $E_{solid} > E_{liquid} > E_{gas}$

10. If $T\uparrow \Rightarrow E\downarrow$

11. $F = \left(\frac{AY}{L}\right)\Delta L$ $[F = (k)x]$ $\Rightarrow k = \frac{AY}{L}$

$PE = \frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{AY}{L}\right)(\Delta L)^2$

$[\tau = c\theta$
$W = \int \tau d\theta = \int c\theta d\theta = \frac{1}{2}c\theta^2]$

12. Poisson's ratio = σ = $\frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\Delta R/R}{\Delta L/L}$

$\boxed{\beta = \frac{Y}{3(1-2\sigma)}}$ ★

$\boxed{\eta = \frac{Y}{2(1+\sigma)}}$

$\boxed{Y = \frac{3\beta - 2\eta}{6\beta + 2\eta}}$

$\boxed{\frac{9}{Y} = \frac{1}{\beta} + \frac{3}{\eta}}$

HYDRO STATICS

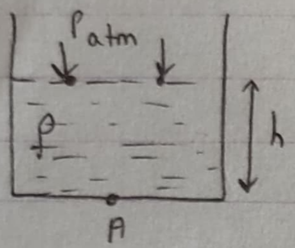
1. $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$.

$\rho_{water} = 1\ kg/L$
$= 1000\ kg/m^3$
$= 1g/cc.$

2. Relative density = $\frac{\text{density of substance}}{\text{density of reference substance.}}$

Specific gravity = $\frac{\text{density of substance}}{\text{density of water}}$.

3.

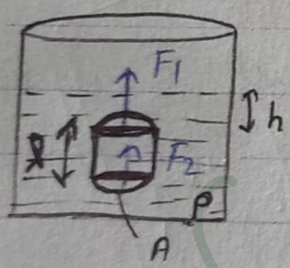


$P_A = P_{atm} + \rho g h.$

(When container is at rest, then at the same horizontal level, pressure will be same).

4. Avg ~~pressure~~ force on side walls of a vessel $= \frac{1}{2} \rho g w h^2$
(w = width, h = height). $\left(P_{avg} = \frac{\rho g h}{2}\right)$

5. Archimedes' Principle – Acts at COM of submerged portion of body.



$F_1 = (P_0 + \rho h g)A$
$F_2 = (P_0 + \rho g h + \rho g l)A$

Al = volume of submerged object.

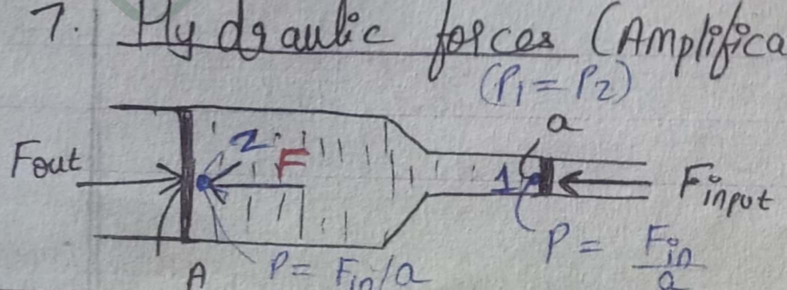
$(F_2 - F_1) = \rho g l A = V\rho g$

Buoyant force = $V\rho g$.

→ Apparent weight $= Mg - V\rho g$.

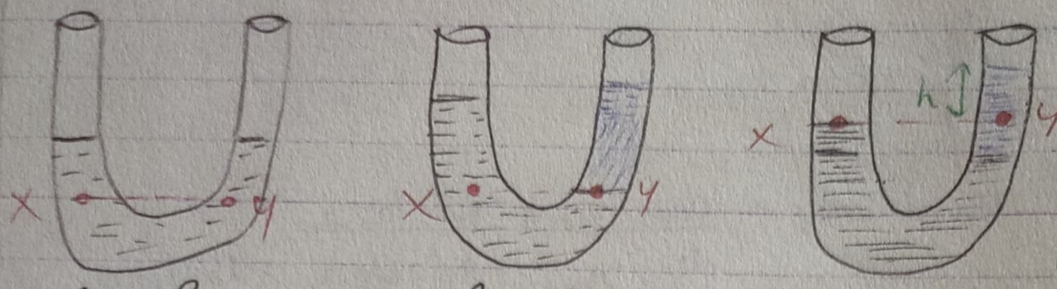
6. Pascal's Law – The pressure applied at one point in an enclosed fluid is transmitted uniformly to every part of the fluid.

7. Hydraulic forces (Amplification of force) $A >>> a$.
$(P_1 = P_2)$



$F_{out} = F = \frac{F_{in}}{a} A$

In the same liquid, same height ⇒ same pressure.



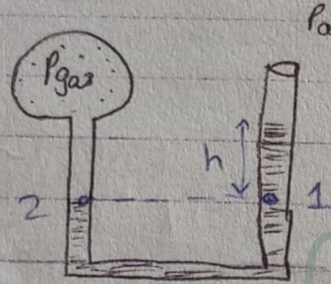
$P_x = P_y$ $\quad$ $P_x = P_y$ $\quad$ $P_x = P_{atm}$

$P_y = P_{atm} + \rho g h$

$\therefore P_y > P_x$.

$P_x \neq P_y$
Height of x, y is same, but liquid is not same.

8. Open Tube Manometer -

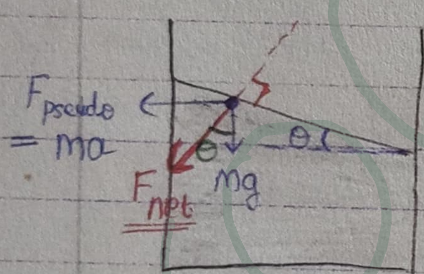


$P_2 = P_1$

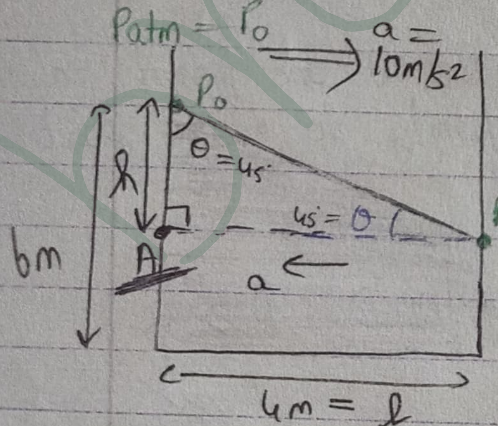
$\Rightarrow P_{gas} = P_{atm} + \rho g h$

9. Pressure In Accelerated Frame

→ (In a static fluid, free surface of fluid is always normal to the effective acceleration acting on it.) AAA



$$\tan\theta = \frac{ma}{mg} = \frac{a}{g}.$$



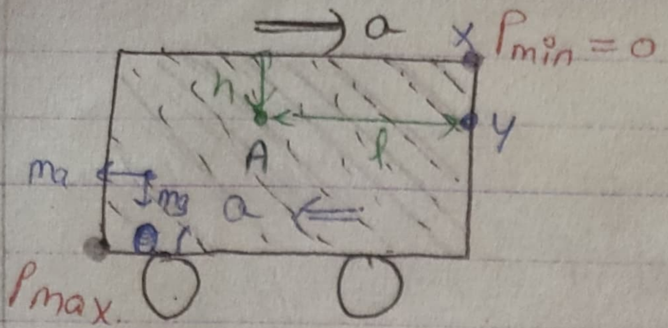
$\tan\theta = \frac{a}{g} = \frac{10}{10} = 1.$ $\quad \therefore \theta = 45°$

$h = 4m, \; l = 4m.$

$$P_A = P_0 + \rho g h = P_0 + \rho a l.$$

Closed Accelerated Container

3D parabola = Paraboloid



$x \to y$

$P_y = \rho g h$

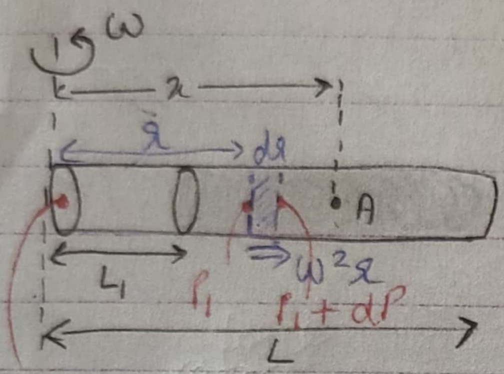
$y \to A$

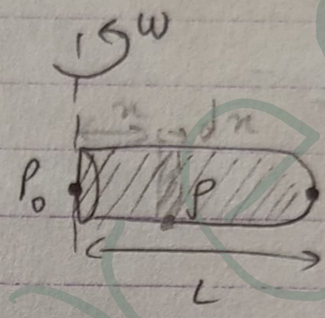
$P_A = P_y + \rho l a$

$= \rho g h + \rho a l$.

$\tan\theta = \frac{a}{g}$

10. Pressure Distribution In Rotating Fluid-filled Tube –





$P = P_0 + \frac{1}{2}\omega^2 L^2 \rho$

P_{atm} x

$\int dP = \int_{L_1} (dx)\rho\, \omega^2 x = P_A$.

$P_A = \frac{1}{2}\omega^2 \rho (x^2 - L_1^2)$.

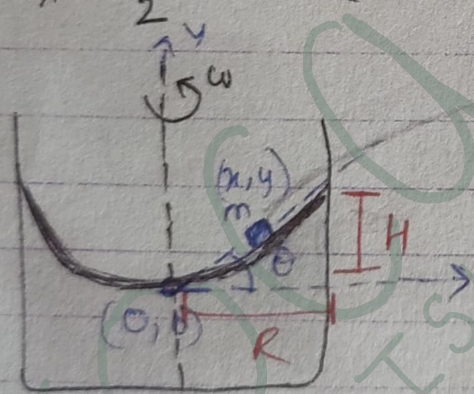
$P_0 \phi = 0$

$P_x = \int_0^L \omega^2 x \rho dx$

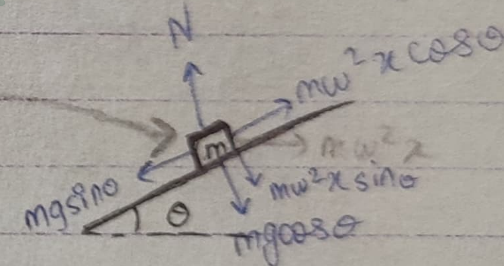
$\omega^2 \rho \frac{1}{2} x^2 \Big|_0^L$

$\omega^2 \rho \frac{1}{2} L^2$

11.



$\tan\theta = \frac{dy}{dx}$.



$\cancel{m}g\sin\theta = \cancel{m}\omega^2 x \cos\theta$

$\tan\theta = \frac{\omega^2 x}{g}$

$\Rightarrow \frac{dy}{dx} = \frac{\omega^2 x}{g}$

$\Rightarrow \int_0^y dy = \frac{\omega^2}{g}\int_0^x x \cdot dx$

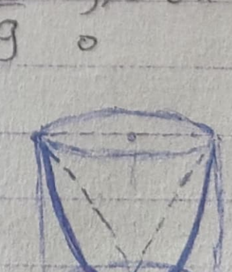
$\therefore \boxed{y = \frac{\omega^2 x^2}{2g}}$

$\therefore \boxed{H = \frac{\omega^2 R^2}{2g}}$.

$(y = H,\ x = R)$

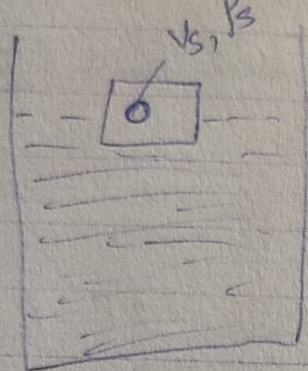
$V_{cone} = \frac{1}{3} V_{cylinder}$

$V_{paraboloid} = \frac{1}{2} V_{cylinder}$



Q: A piece of ice having a stone frozen in it floats in a glass filled with water. How will the level of water in the vessel change when the ice melts.

Ans



$V_i = V_{inside}$

At equillibrium:-

$(m_{ice} + m_s)g = V_i \rho_w g$

$\Rightarrow V_i = \frac{m_{ice}}{\rho_w} + \frac{m_s}{\rho_w}$

On melting - Vol. of water from ice $= \frac{m_{ice}}{\rho_w} = V_1$

Vol. of stone $= \frac{m_s}{\rho_s} = V_2$

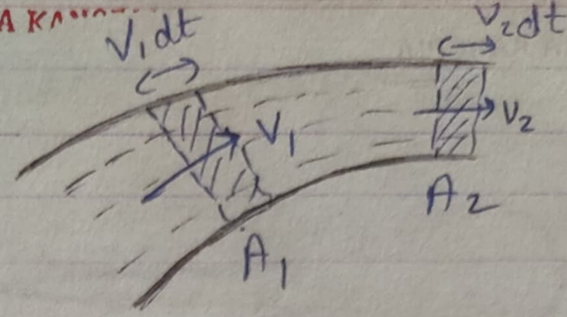
$\rho_s > \rho_L$

$V = V_1 + V_2$ $\quad \therefore V_1 + V_2 < V_i$

$\therefore$ level will decrease.

HYDRO DYNAMICS

1. **Continuity equation** –



$dm_1 = dm_2$

$\rho_1 A_1 V_1 dt = \rho_2 A_2 V_2 dt$

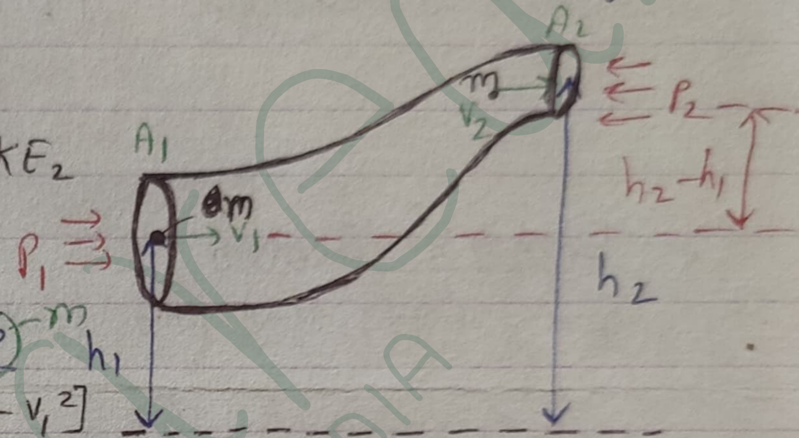
(for incompressible fluids, $\rho_1 = \rho_2$).

$\therefore \boxed{A_1 V_1 = A_2 V_2}$.

2. **Bernoulli's Theorem** – (Energy conservation for fluid).
(applicable for stream-line, non-turbulent, non-viscous flow)

Work-energy theorem –

$W_{g_1} + W_{P_1} + KE_1 = W_{g_2} + W_{P_2} + KE_2$



$\Rightarrow -mg(h_2 - h_1) + \dfrac{P_1(A_1 V_1 dt\, \rho)}{\rho} - \dfrac{P_2(A_2 V_2 dt\, \rho)}{\rho} = \dfrac{1}{2} m [V_2^2 - V_1^2]$

mass = m

$(W_{P_1} = P_1 A_1 V_1 dt)$

$\Rightarrow -g(h_2 - h_1) + \dfrac{P_1}{\rho} - \dfrac{P_2}{\rho} = \dfrac{1}{2}[V_2^2 - V_1^2]$

$\Rightarrow -\rho g(h_2 - h_1) + P_1 - P_2 = \dfrac{1}{2}\rho[V_2^2 - V_1^2]$

$\Rightarrow \underbrace{\rho g h_1}_{\text{PE density}} + \underbrace{\dfrac{1}{2}\rho V_1^2}_{\text{KE density}} + \underbrace{P_1}_{\text{Pressure energy density}} = \rho g h_2 + \dfrac{1}{2}\rho V_2^2 + P_2$

$\Rightarrow \boxed{P + \rho g h + \dfrac{1}{2}\rho V^2 = \text{constant}}$

$\boxed{\dfrac{P}{\rho g} + h + \dfrac{1}{2}\dfrac{V^2}{g} = \text{constant}}$

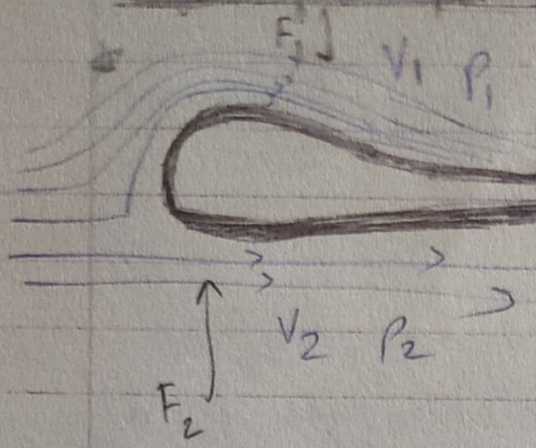
$\dfrac{P}{\rho g}$ – Pressure head

h – gravitational head / Potential head

$\dfrac{V^2}{2g}$ – KE head / Velocity head

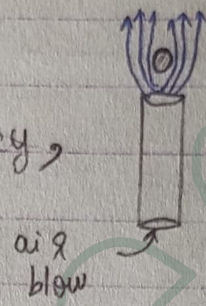
☆ More dense lines => more velocity

3. Lift of aeroplane (Based on Bernoulli's theorem)



$V_1 > V_2$
$\therefore P_1 < P_2$
$\therefore$ upward force (F_2) > downward force (F_1)
$\therefore$ lift of aeroplane.

Other applications - Atomizer spray,



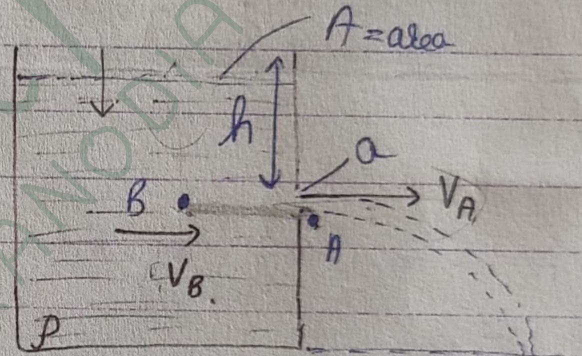
, Venturimeter.

4. Toricelli's Theorem

→ efflux velocity of a fluid flowing out through a hole in a container.

$A >>> a$
$\therefore V_B \approx 0$

$\begin{cases} P_A = P_{atm} \\ P_B = P_{atm} + h\rho g \end{cases}$



$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$

$\frac{1}{2}\rho V_A^2 = h\rho g$

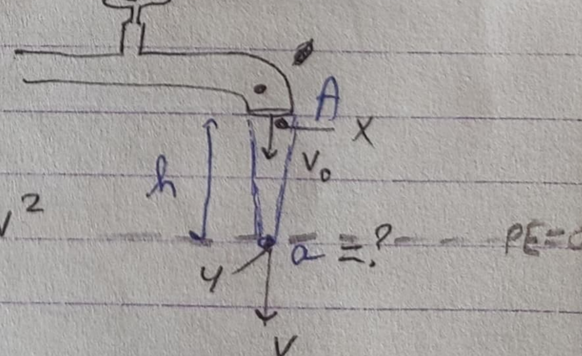
$\boxed{V_A = \sqrt{2gh}}$

5. Free-falling liquid -

According to Bernoulli's theorem -

at (x) and (y) -

$P_{atm} + \frac{1}{2}\rho V_0^2 + \rho g h = P_{atm} + 0 + \frac{1}{2}\rho V^2$



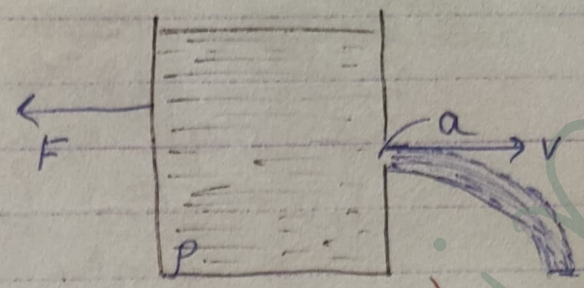
$\Rightarrow V^2 = V_0^2 + 2gh.$

Equation of continuity - $V_0 A = V a$

$\Rightarrow a = \left(\frac{V_0}{V}\right)A = \frac{V_0 A}{\sqrt{V_0^2 + 2gh}}.$

Reaction Force Due To Ejection of A Fluid -

$F = \frac{dP}{dt} = \Delta P_{1\,sec}$



In 1 sec -

mass of water ejected $= m = \rho(av)$

$\therefore \Delta P = mv$

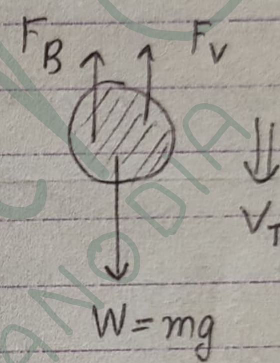
$= \boxed{\rho a v^2 = F}$

7. Viscosity (η)

$\rightarrow F_v = \eta A \left(\frac{dv}{dy}\right)$ velocity gradient.

units of η -
SI unit = Poiseuille (Pl)
CGS = Poise.
1 Pl = 10 poise.

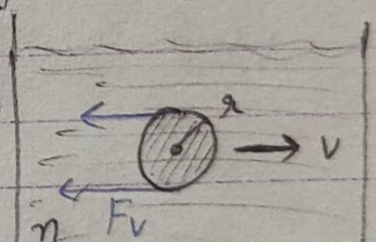
8. Stoke's Law



as V increases downward,
F_v also keeps ↑.
Finally $F_B + F_V = W$ ∴ no net force
⇒ velocity becomes constant
$= V_T$ (terminal velocity).

Initially -
$W > F_v$
∴ a ⇓
but F_v keeps ↑.
Then - $W = F_v$

$\boxed{F_v = 6\pi\eta r V_T}$ → applicable only in homogeneous medium at rest.

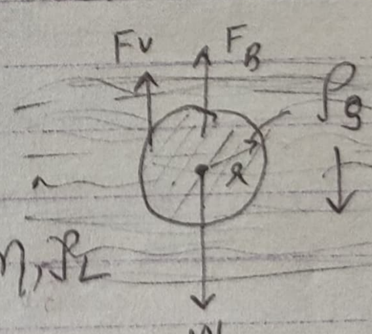


Q. $V_T = ?$

$F_v + F_B = W$

$\Rightarrow 6\pi\eta r V_T + \frac{4}{3}\pi r^3 \rho_L g = \rho_S \frac{4}{3}\pi r^3 g$

$$\boxed{V_T = \frac{2}{9} r^2 \left[\frac{\rho_S - \rho_L}{\eta}\right] g.}$$



9. Critical Velocity and Reynold's Number (N_R)

→ upto a certain velocity, fluid flow remains streamline and beyond critical velocity (V_C), flow becomes turbulent.

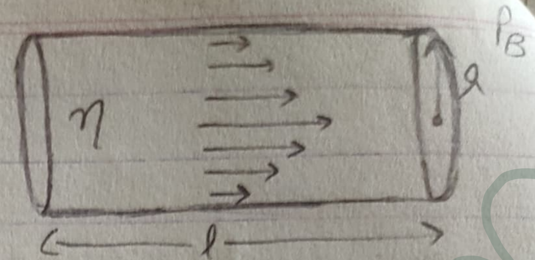
$$\boxed{V_C = \frac{N_R\,\eta}{r\,\rho}}$$

r = radius of pipe.

★★ If Medium is not at rest,
Insted of V_T take V_{rel}
where V_{rel} = relative velocity of object wrt medium

$(Q = A_1 v_1 = A_2 v_2 = \text{flow})$

10. Poiseuille's Equation



A fluid always flows from high to low pressure. $(P_A > P_B)$

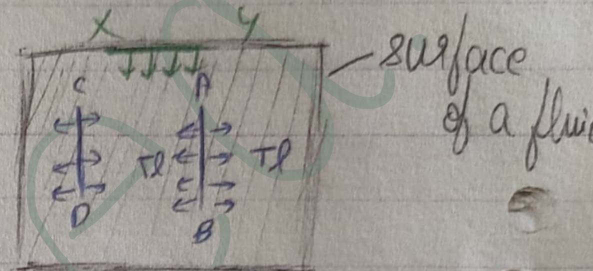
$$\boxed{Q = \frac{\pi r^4}{8 \eta L}(\Delta P)}$$

11. Surface Tension (T) – (ST)

$$\boxed{T = \frac{F}{l}.}$$

on pricking a thread (loop) in soap film, it becomes circularized.

On XY, force is not balanced.



no net force on AB and CD

→ ST ↓ as T ↑ and it becomes zero at critical temperature

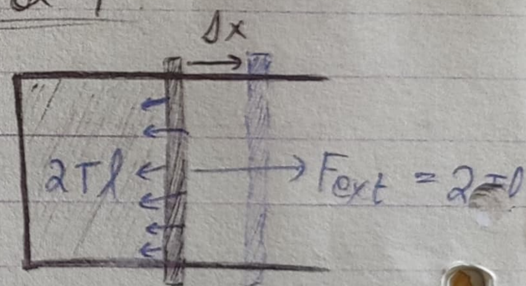
→ For insoluble impurities ⇒ ST ↓

soluble impurities ⇒ ST ↓ or ↑.

Work done $= W = 2Tl(\Delta x)$.

$= T(2l\Delta x)$

$= T\Delta S = \Delta U$.



$\therefore T = \frac{F}{l} = \frac{\Delta U}{\Delta S}$

ΔS = change in surface area.

→ merging of small drops ⟶ big drop

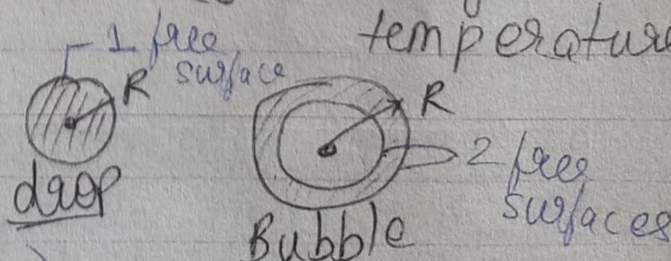
$\Delta U = ms\,\Delta\theta$

($\Delta\theta$ = change in temperature)

12. Excess Pressure –

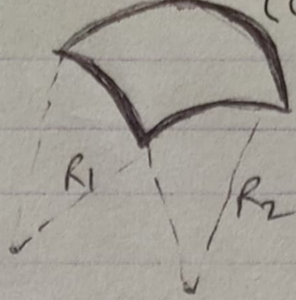
→ $\Delta P = P_{in} - P_{out} = \frac{2T}{R}$ (drop)

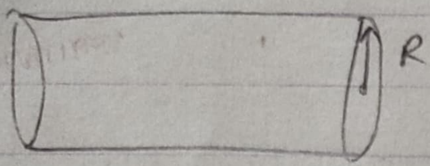
$\Delta P = P_{in} - P_{out} = \frac{4T}{R}$ (bubble)



If $\theta = 0°$ $(\cos\theta = 1)$ $R = r$

$$P_{excess} = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$





$$P_{excess} = \frac{T}{R}$$

13. Angle of Contact / Wetting Angle – (θ)

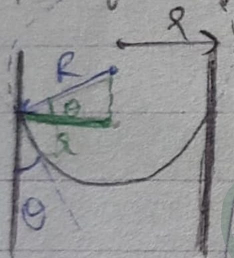
→ If $\theta \downarrow$
⇒ good wetting

→ $\vec{F}_R = \vec{F}_C + \vec{F}_A$ (C = cohesive, a = adhesive, R = resultant)

→ If $\theta < 90°$
⇒ $F_a > F_c$
Eg – water glass pair
⇒ concave meniscus

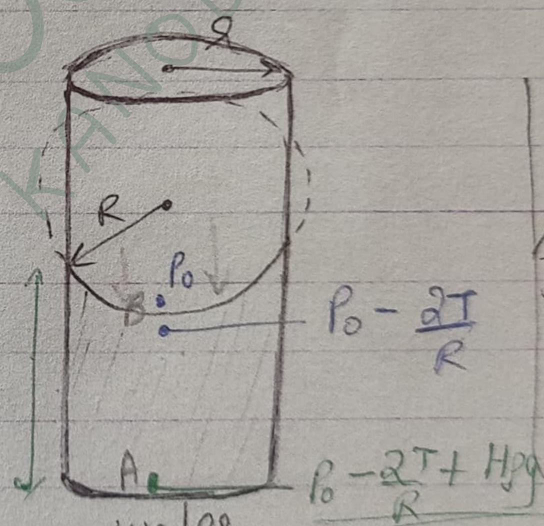
If $\theta > 90°$
⇒ $F_c > F_a$
Eg – mercury glass pair.
⇒ convex meniscus.

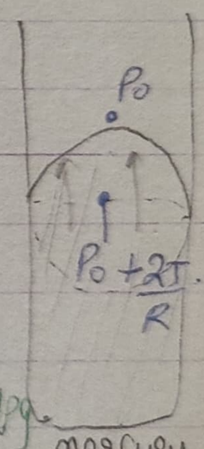
14. Shape of Liquid Meniscus –



$R\cos\theta = r$

If $\theta = 0$
⇒ $R = r$



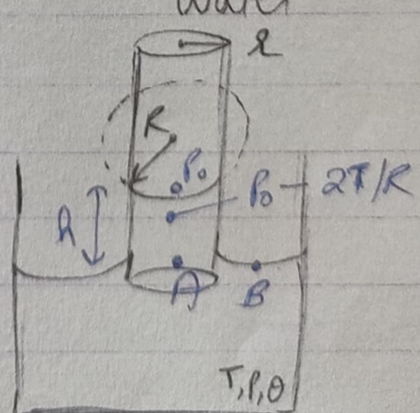


$P_A = P_0 - \frac{2T}{R} + h\rho g$

$P_B = P_0$

$P_A = P_B$

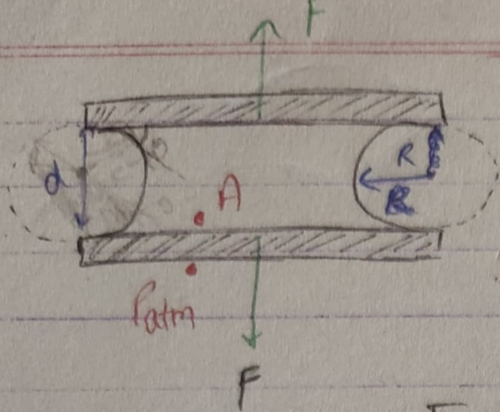
⇒ $\cancel{P_0} - \frac{2T}{R} + h\rho g = \cancel{P_0}$



⇒ $h\rho g = \frac{2T}{R}$ ⇒ $h = \frac{2T}{R\rho g}$ → $hR = \frac{2T}{\rho g}$ = constant

$R\cos\theta = r$. ∴ $\boxed{h = \frac{2T\cos\theta}{r\rho g}}$

∴ $hR = h'R'$ = constant.

15. 

Liquid Between Two Parallel Plates

$R\cos\theta = d/2$ (θ = angle of contact)

cylindrical

$\therefore P_A = P_{atm} - \frac{T}{R}$

$$F = \left(P_{atm} - P_{atm} + \frac{T}{R}\right)A$$

$$\Rightarrow F = \frac{TA}{R} = \frac{2TA\cos\theta}{d}$$