OSCILLATIONS

1. **Periodic Motion** ⇒ motion which repeats continuously after a regular interval of time.

→ **Time period (T)** = regular interval of time.

→ **Frequency (ν)** = No. of cycles/complete motion per unit time. (unit = Hz or s^{-1})

$$\boxed{\nu = \frac{1}{T}}$$

2. **Oscillatory/Vibratory Motion** ⇒ periodic motion in which a particle moves to and fro about a fixed position.

→ **Mean position** ⇒ fixed position ⇒ point of stable equilibrium

Types of Oscillations

Free Oscillations
- no loss of energy
- constant amplitude (In ~~air~~ vacuum)

Damped Oscillations
- loss of energy
- decreasing amplitude (In ~~vacuum~~ air)

Forced Oscillations
- presence of external force
- constant / varying amplitude

3. **Harmonic motion** ⇒ trigonometric fun^n of constant amplitude and single frequency.

$y = A\sin\omega t$, $y = A\cos\omega t$ } Harmonic functions

tan, cot, sec, cosec } Non-harmonic functions

$F = \boxed{-Kx^n}$ $\quad n = 1, 3, 5, 7, \ldots\ldots$

If $n = 1$ ⇒ SHM ($F = -Kx$)

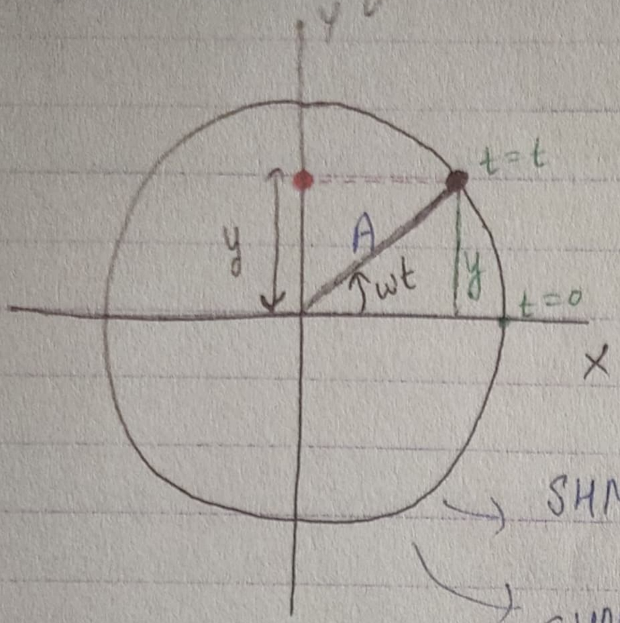
4. **Simple Harmonic Motion (SHM)** ⇒ oscillatory motion in which the acceleration of the particle is directly proportional to the displacement of the particle from its mean position.

$\vec{a} \propto -\vec{x}$ ⇒ SHM $\quad \left\{ \begin{array}{l} F = -Kx \\ ma = -Kx \end{array} \right\}$

UCM = uniform circular motion

5. Representation of SHM

• UCM • SHM



radius of circle = amplitude of SHM

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$

SHM in y-direction ⇒ $\boxed{y = A\sin\omega t}$

SHM in x-direction ⇒ $\boxed{x = A\cos\omega t}$

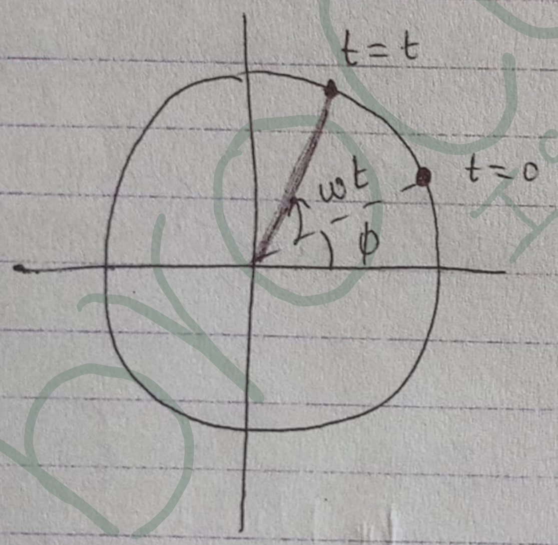
6. $y = A\sin\omega t$

$v = \frac{dy}{dt} = A\omega\cos\omega t$

$a = \frac{dv}{dt} = -A\omega^2\sin\omega t = -\omega^2 y$

$\boxed{a = -\omega^2 y}$ $\therefore a \propto -y$.

7.



ϕ = initial phase.

$$\boxed{y = A\sin(\omega t + \phi)}$$

$(\omega t + \phi)$ = phase.

8. $y = A\sin(\omega t + \phi)$

$$\boxed{v = A\omega\cos\omega t = \omega\sqrt{A^2 - y^2}}$$

$\therefore \sin(\omega t + \phi) = \frac{y}{A}$

$\therefore \cos(\omega t + \phi) = \sqrt{1 - \sin^2(\omega t + \phi)}$

$= \sqrt{1 - \frac{y^2}{A^2}}$

ω = angular frequency

→ $V_{max} = \omega A \Big|_{y=0}$

$V_{min} = 0 \Big|_{y=A}$

→ $a = \frac{dv}{dt} = -A\omega^2 [\sin(\omega t + \phi)] = -\omega^2 y$

$a_{max} = \pm \omega^2 A \Big|_{y=A}$

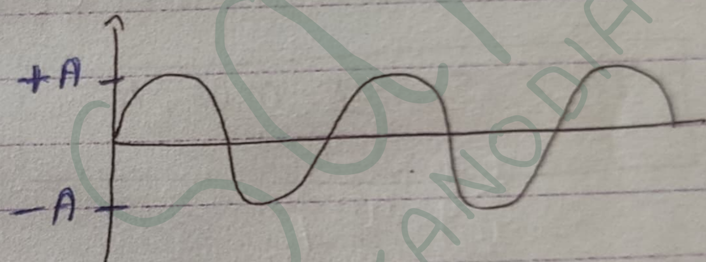
$a_{min} = 0 \Big|_{y=0}$

→ $a = \frac{d^2y}{dt^2} = -\omega^2 y$

$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$ ⇒ differential equation of SHM.

used to check if given eq^n is eq^n of SHM.

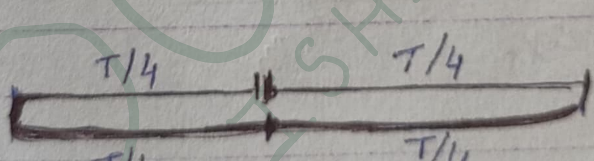
9. $y = A\sin(\omega t + \phi)$



10. $360° \longrightarrow T$

$\therefore 90° \longrightarrow T/4$

If $y = A\sin\omega t \Rightarrow$ At $90°$, $y = A$.



11.

$-A$	0	$+A$
$(v=0)$	$PE=0$	$(v=0)$
$KE=0$	$(x=0)$	$KE=0$
$PE=max$	$KE=max$	$PE=max$
(x_{max})	(V_{max})	(x_{max})

At $x=0$,

$PE = 0$

$KE = \frac{1}{2} m (V_{max})^2$

$= \frac{1}{2} m\omega^2 A^2$

At $x=A$, $E_T = \frac{1}{2} kx^2$

$= \frac{1}{2} kA^2$

$\therefore \boxed{E_{Total} = \frac{1}{2} m\omega^2 A^2}$

$\therefore m\omega^2 = k$.

12\. $V(t) = A\omega\cos(\omega t + \phi)$ $\quad V(y) = \omega\sqrt{A^2 - y^2}$.

$\rightarrow$ $KE(t) = \frac{1}{2} m A^2 \omega^2 \cos^2(\omega t + \phi)$.

$\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = 1/2$

$\langle \sin\theta \rangle = \langle \cos\theta \rangle = 0$.

$\rightarrow$ $\boxed{\langle KE(t) \rangle = \frac{1}{4} m A^2 \omega^2} = \frac{E_T}{2}$.

$\rightarrow$ $KE(y) = \frac{1}{2} m \omega^2 [A^2 - y^2]$.

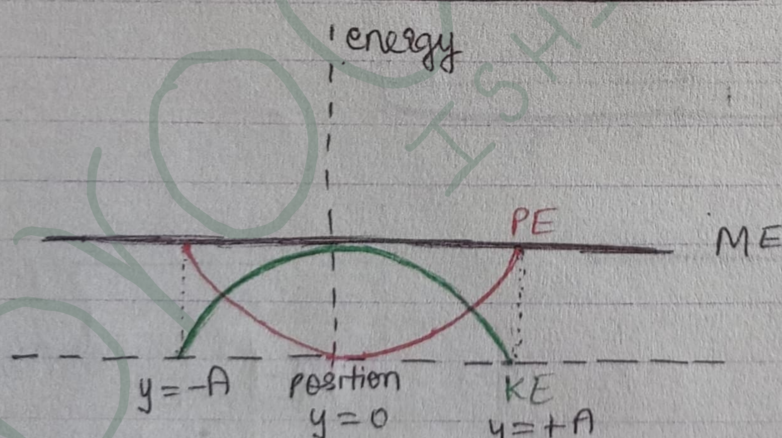
$y = A\sin(\omega t + \phi)$

$\rightarrow$ $PE(t) = \frac{1}{2} k y^2 = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + \phi)$

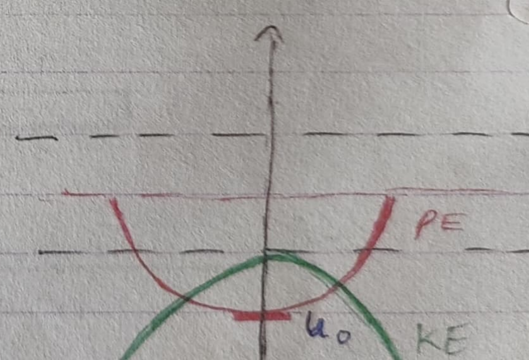
$\rightarrow$ $\therefore \boxed{\langle PE \rangle = \frac{1}{4} m\omega^2 A^2} = \frac{E_T}{2}$.

If at mean position, $PE = PE_0$

$\Rightarrow \boxed{E_T = PE_0 + \frac{1}{2} m\omega^2 A^2}$.



When $U = 0$ ($PE = 0$) at mean position



When $PE = U_0$ at mean position.

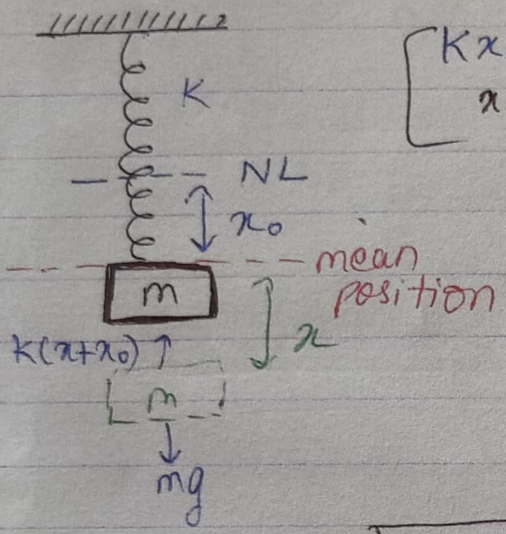
natural length $a_r = a_{restoring}$

$\boxed{m\omega^2 = k} \Rightarrow \omega = \sqrt{\frac{k}{m}}$

$T = \frac{2\pi}{\omega}$

$\therefore \boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$

14. Time period of a Spring Block System



$\begin{bmatrix} Kx_0 = mg \\ x_0 = \frac{mg}{K} \end{bmatrix}$

$F_{net} = K(x+x_0) - mg$

$\Rightarrow ma_r = Kx + Kx_0 - mg$

$= Kx + \cancel{mg} - \cancel{mg}$

$\Rightarrow ma_r = Kx$

$\therefore a_r \propto x \Rightarrow$ SHM.

$\therefore g$ can be ignored

$m\omega^2 \cancel{x} = K\cancel{x}$

$\therefore \omega = \sqrt{\frac{K}{m}}$

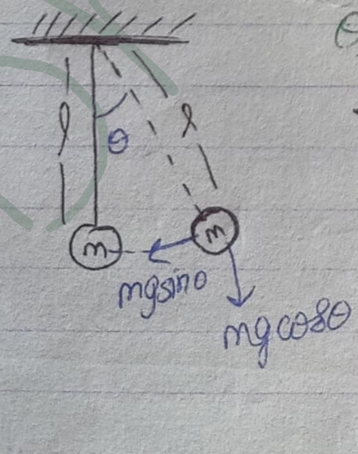
$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}}$

15. $a = -\omega^2 y \Rightarrow$ linear SHM

$\boxed{\alpha = -\omega^2\theta} \Rightarrow$ angular SHM

$\boxed{\frac{d^2\theta}{dt^2} + \omega^2\theta = 0}$

16. Time period of a Simple Pendulum



θ is very small. $(\because \sin\theta \approx \theta)$

$\tau_r = mg\sin\theta\,(l) = I\alpha_r$

$\Rightarrow \cancel{m}g\sin\theta \cdot \cancel{l} = (\cancel{m}l^{\cancel{2}})\alpha_r$

$\Rightarrow \alpha_r = \frac{g}{l}\sin\theta = \frac{g}{l}\theta = \omega^2\theta$

$\therefore \omega^2 = \frac{g}{l}$

$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}}$

17. ## Torsional Pendulum

$$\boxed{\tau_R = -C\theta}$$

C = coefficient of stiffness
= torsional coefficient.

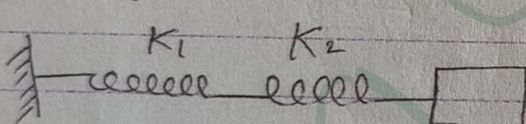
$\tau_R = C\theta = I\alpha_R \Rightarrow \alpha_R = \left(\frac{C}{I}\right)\theta$

$\because \alpha_R \propto \theta$
$\alpha_R = \omega^2\theta.$

$\therefore \boxed{\omega = \sqrt{\frac{C}{I}}}$

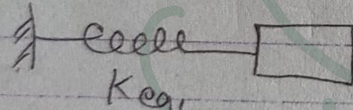
$\therefore \boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}}$

18. ## Spring Combinations in SHM -

→ 

$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}}$ ⇒ series

⇓



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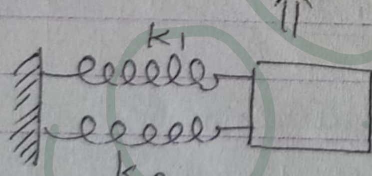
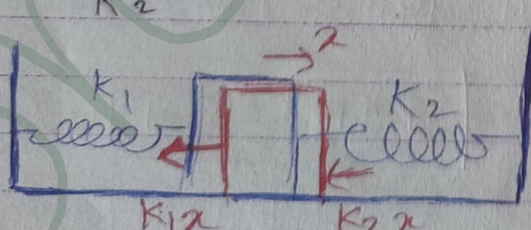
→ 

$\boxed{K_{eq} = K_1 + K_2}$ ⇒ parallel



⇒ parallel

$\therefore F_{net} = (K_1 + K_2)x = ma_x$
$= m\omega^2 x$

1. **Effect of Constant External Force on SHM**

Time period does not change.
Only mean position changes.

20. **Energy Method To Finding Frequency (ν, ω, T) of SHM**

Steps -

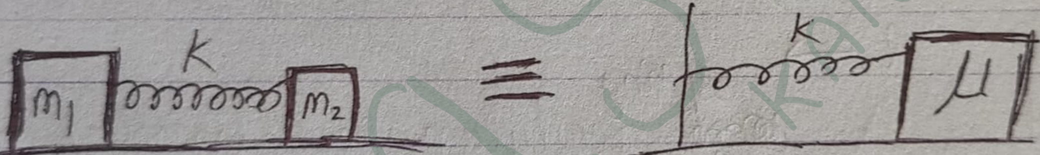
① Find equillibrium position by balancing the forces/ torques acting on body.

② Displace the body and at an intermediate position, write the total energy of the system
(take zero PE at mean position)

③ Differentiate the equation of Total energy and equate it to zero

$$\left(\because E_T = \text{constant} \quad \therefore \frac{dE_T}{dt} \text{ or } \frac{dE_T}{dx} = 0. \right)$$

21. **Time Period of SHM of 2 Blocks - spring system**



$$\boxed{\mu = \text{reduced mass} = \frac{m_1 m_2}{m_1 + m_2}} \quad \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}.$$

$$\boxed{T = 2\pi \sqrt{\frac{\mu}{k}}}$$

22. **Superposition of 2 SHMs**

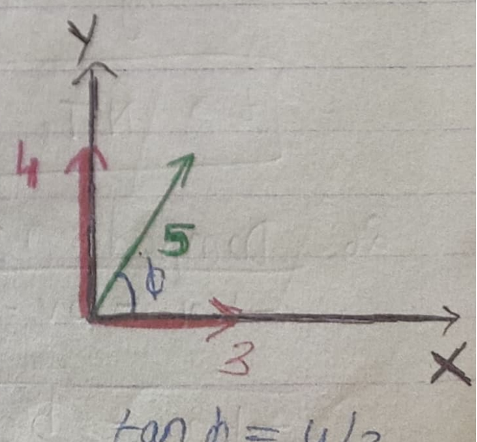
Phasor

$y_1 = 3 \sin \omega t$

$y_2 = 4 \cos \omega t = 4 \sin\left(\omega t + \frac{\pi}{2}\right)$

$y = y_1 + y_2 = 5 \sin(\omega t + \phi)$

$= 5 \sin(\omega t + 53^\circ)$.



$\tan \phi = 4/3$

$\therefore \phi = 53^\circ$

(a) <u>To convert given eq^n into eq^n of SHM -</u>

$y = 3\sin\theta + 4\cos\theta$

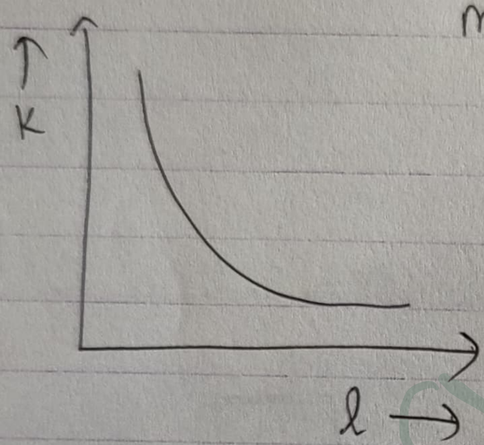
$= 5\left(\frac{3}{5}\sin\theta + \frac{4}{5}\cos\theta\right)$ $\quad \sqrt{3^2+4^2} = 5$

$= 5(\cos 53^\circ \sin\theta + \cos\theta \sin 53^\circ)$

$\underline{y = 5\sin(\theta + 53^\circ)}$

(23) $\vec{F} = -K\vec{x}$

K depends on length (l), radius (r), material of wire used in spring.



$\boxed{kl = \text{constant}}$

24. Time period of a ~~simple~~ spring pendulum is independent of a_{dg}

→ If length of the spring is made n times, then k becomes $\frac{1}{n}$ times and T becomes $\sqrt{n}$ times.

→ If a spring (K) is divided into n equal parts, then spring constant of each part becomes nK and T becomes $\frac{1}{\sqrt{n}}$ times.

25. Time (minimum) taken for a long and short pendulum oscillating simultaneously to be in same phase -

$$\boxed{t = NT_l = (N+1)T_s}$$

26. <u>Damped Oscillations</u> -

retarding force $\vec{F} = -b\vec{v}$ $\quad$ b = damping coeffecient

$\vec{F} = -K\vec{x} \Rightarrow$ restoring force.

$-Kx - bv = ma.$

$$\boxed{-Kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}}$$

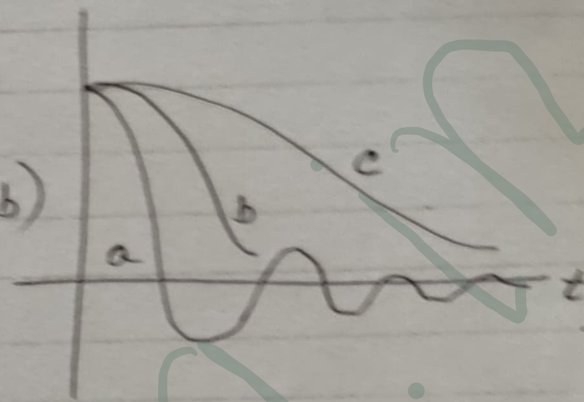
$$\boxed{x = A e^{-bt/2m}\cos(\omega' t + \phi)}$$

$$\boxed{\omega' = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}}$$

→ when $\frac{b}{2m} < \omega \Rightarrow$ underdamped (a)

$\frac{b}{2m} = \omega \Rightarrow$ critically damped (b)

$\frac{b}{2m} > \omega \Rightarrow$ over-damped (c)



→ $$\boxed{E(t) = \frac{1}{2} k A^2 e^{-bt/m}}$$

27. <u>Forced Oscillations</u>

→ system does not oscillate with natural frequency (ω), but with driven frequency (ω_d)

→ External force $F(t)$ of amplitude F_0 –

$$F(t) = F_0 \cos \omega_d t$$

$$m a(t) = -kx - bv + F_0 \cos \omega_d t$$

$$\boxed{m \frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos \omega_d t}$$

$$\boxed{x = A' \cos(\omega_d t + \phi)}$$

$$\boxed{A' = \frac{F_0}{m\sqrt{(\omega_d^2 - \omega^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}} \quad \left(\omega = \sqrt{\frac{k}{m}}\right).$$

→ For resonance, $\omega_d \approx \omega$.