GRAVITATION

1. $\vec{F} = \frac{GM_1M_2}{r^2} = \frac{GM_1M_2}{r^3}\vec{r} = \frac{GM_1M_2}{r^2}\hat{r}$.

$G = 6.67 \times 10^{-11}\ Nm^2/Kg^2$

- always attractive in nature
- central force
- long range force.

superposition applicable

2. Gravitational Field Intensity (I) $\boxed{\vec{I} = \frac{\vec{F_g}}{m}}$

Superposition is applicable.

- Point where $I_{net} = 0 \Rightarrow$ Null point

Intensities Due To Different Bodies

(a) Uniform line	$I_x = \frac{G\lambda}{R}[\sin\theta_1 + \sin\theta_2]$ $I_y = \frac{G\lambda}{R}[\cos\theta_2 \sim \cos\theta_1]$
(b) Uniform ring	$I_{axis} = \frac{GMx}{(R^2+x^2)^{3/2}}$ $\left\{x = \frac{R}{\sqrt{2}} = \text{maxima}\right\}$
(c) Spherical shell	$I_{centre} = 0$ $I_{out} = \frac{GM}{r^2}$ $I_{in} = 0$
(d) Solid sphere	$I_{out} = \frac{GM}{r^2}$ $I_{in} = \frac{GMr}{R^3}$.

3. $V_P = -\int_{\infty}^{P} \vec{I}\cdot d\vec{r}$ $V_{1-2} = V_2 - V_1 = -\int_{r_1}^{r_2} \vec{I}\cdot d\vec{r}$

→ Point Mass - $V = -\frac{GM}{r}$.

→ Ring - $V_{axis} = -\frac{GM}{\sqrt{R^2+x^2}}$.

→ Shell - $V_{out} = -\frac{GM}{r}$ $V_{in} = -\frac{GM}{R}$.

→ Solid Sphere - $V_{out} = -\frac{GM}{r}$ $V_{in} = -\frac{GM}{2R^3}[3R^2 - r^2]$

4. $U_p = mV_p$

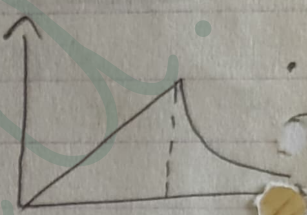
→ U is the work done to make the system.

$U_{system} = \dfrac{U_1 + U_2 + U_3 + \cdots U_n}{2}$

5. For earth. $R_{eq} > R_{poles}$ $(R_{eq} - R_{poles} = 21\ km)$

→ Adg
Acceleration due to gravity - $g = I$

$g_{in} = \dfrac{GM_E r}{R^3}$ $g_{out} = \dfrac{GM_E}{r^2}$



$g_0 = g_{surface} = \dfrac{GM_E}{R_E^2} = 9.8\ m/s^2$

6. Variation of g -

→ $g_{in} = g_0 \left(1 - \dfrac{d}{R_E}\right)$ d = depth from surface of earth
$g_0 = g_{surface}$

$\therefore \Delta g\% = -\dfrac{d}{R_E} \times 100$.

→ $g_{out} = g_0 \left(1 - \dfrac{2h}{R_E}\right)$ h = height from surface of earth

$\therefore \Delta g\% = \dfrac{-2h}{R_E} \times 100$.

7. Effect of Rotation on g

$\boxed{g_{eff} = g_0 - \omega^2 R_E \cos^2\theta.}$ θ is measured from the equator.

Escape velocity ⇒ minimum required velocity to escape earth's gravitational field.

$$\boxed{V_E = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E} = 11.2\ km/s.}$$

$$\boxed{V_P^2 = V_R^2 + V_e^2}$$

2. Kepler's Laws of Planetary Motion

→ 1st Law – Areal velocity (rate at which area is swept) of the planet revolving around the sun is constant and is $= \frac{L_s}{2m_p}$.

$$\boxed{A = \frac{L_s}{2M_p} T} \qquad \boxed{\pi ab = \frac{L_s}{2M_p} T.}$$

→ $r_{max} = a(1+e) \Rightarrow$ apogee / aphelion $\Rightarrow V_{min}$

$r_{min} = a(1-e) \Rightarrow$ perigee / perihelion. $\Rightarrow V_{max}$

$$\frac{r_{max}}{r_{min}} = \frac{V_{max}}{V_{min}} = \frac{1+e}{1-e}.$$

$$V_{max} = \sqrt{\frac{GM_s}{a}\left(\frac{1+e}{1-e}\right)} \qquad V_{min} = \sqrt{\frac{GM_s}{a}\left(\frac{1-e}{1+e}\right)}.$$

independent of mass of planet (M_p).

$$\boxed{ME = -\frac{GM_sM_p}{2a}} \qquad \boxed{KE = \frac{GM_sM_p}{2a}} \qquad \boxed{PE = -\frac{GM_sM_p}{a}}.$$

→ Law of Time period – $\boxed{T^2 \propto r^3}$ (circle) $\boxed{T^2 \propto a^3}$ (ellipse).

$$T^2 = \frac{4\pi^2 a^3}{GM_s}.$$

EARTH ROTATES FROM WEST TO EAST

10. Satellite - anything revolving around the earth (planet).

natural — artificial

moon.

For satellites - only that orbit is allowed whose focus/centre coincides with the centre of the earth.

→ Orbital Velocity and Energy of Earth's Satellite -

$$\boxed{V_o = \sqrt{\frac{GM_e}{R_E}} = \frac{V_{escape}}{\sqrt{2}}} \qquad \boxed{T = \frac{2\pi r^{3/2}}{\sqrt{GM_e}} = 84.6 \text{ min}}$$

$< \sqrt{gR_E}$.

$$ME = -\frac{GM_e m}{2r}.$$

11. Work done by external agent in changing orbital radius from r_1 to r_2

$$W_{ext} = \frac{GM_e M_e}{2}\left[\frac{1}{r_1} - \frac{1}{r_2}\right].$$

12. Geostationary Satellite - (designed for places on the Equator).
(at rest wrt to earth)

$\boxed{T = 24 \text{ h.}}$

→ in equatorial plane, at radius of 36000 km = 5.62 R_E from earth's surface

at radius of 43000 km = 6.62 R_E from earth's centre.

13. Polar Satellite - (For predicting weather)
→ passing through earth's axis.
→ covers the whole surface of the earth.

generally - radius = 500 - 800 km from surface.

14. Communication Range

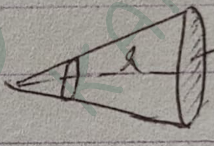
$$\boxed{(R+h)\cos\theta = R.}$$

$$\boxed{\text{Area} = 2\pi R^2[1-\cos\theta]}$$

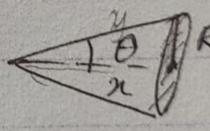
15. Weightlessness in Satellite — $N = 0$ weightlessness.

16. $\left\{ \underset{\text{velocity}}{V} \propto r^{\frac{1-n}{2}} \qquad \underset{\text{time period.}}{T} \propto r^{\frac{n+1}{2}} \right\}$ where $F \propto \frac{1}{r^n}$.

17. Solid Angle –



$$\boxed{d\Omega = \frac{ds}{r^2}}$$



$y \sin\theta = R$

$y \cos\theta = x$

$$\boxed{\Omega = 2\pi(1-\cos\theta)}$$

$$\boxed{\text{Area} = 2\pi R^2(1-\cos\theta)}$$

$\Omega = \frac{2\pi \cancel{R^2}}{\cancel{R^2}} \sin^2\theta$

$4\pi R^2 \longrightarrow 2\pi \longrightarrow 4\Omega$

$\Omega = 2\pi(1-\cos^2\theta)$