EMI

1. **Magnetic flux** $\boxed{\phi = N\vec{B}\cdot\vec{A}}$ $(1\,Wb = 10^8\,Mx)$

2. **Faraday's first law** ⇒ When ϕ changes with time or field lines are cut by a conducting wire, emf is induced.
Faraday's second law ⇒ $\varepsilon = -\frac{d\phi}{dt}(N)$.

3. **Lenz's law** ⇒ Induced current produced in a circuit always flows in such a direction that it opposes the change or cause that produced it.

$$\boxed{\varepsilon = -N\frac{d\phi}{dt}} \qquad \boxed{I = \frac{\varepsilon}{R}}$$

$$\boxed{H = \int I^2 R\,dt = \int \frac{\varepsilon^2}{R}\,dt}$$

$$\boxed{q = \int I\,dt = \int \frac{\varepsilon}{R}\,dt = \int \frac{d\phi}{dt}\times\frac{1}{R}\times dt = \frac{\Delta\phi}{R}}$$

4. **Induced electric field** ⇒ $\boxed{\varepsilon = -N\frac{d\phi}{dt} = \oint \vec{E}_{in}\cdot d\vec{l}}$

Direction of E_{ind} is same as the direction of current. non-conservative electric field.

- field lines are always closed curves.
- when a unit charge goes around the loop, the total work done = emf in loop.

$E_{inside} = \frac{r}{2}\left(\frac{dB}{dt}\right)$ $\qquad E_{outside} = \frac{R^2}{2r}\left(\frac{dB}{dt}\right)$

$E_{surface} = \frac{R}{2}\left(\frac{dB}{dt}\right)$.

5. **Self-inductance (L)** - $\boxed{\begin{array}{l} N\phi = \phi_T = LI. \\ \varepsilon = -N\frac{d\phi}{dt} = -L\frac{dI}{dt}. \end{array}}$

Solenoid $\boxed{L = \mu_0 n^2 A l = \mu_0 n^2 V.}$

6. **Energy in an inductor** - $\boxed{P = ie = i\left(L\frac{di}{dt}\right)}$.

$\boxed{U = \int i L\,di = \frac{1}{2}LI^2}$

Ideal solenoid $\boxed{\frac{U}{V} = \frac{B^2}{2\mu_0}}$.

energy density.

7. <u>Growth of current</u> ⇒ $I = I_0(1 - e^{-t/\tau})$

$\tau = \frac{L}{R}$ $\left(I_0 = \frac{\varepsilon}{R}\right) \Rightarrow$ at $t = \infty$

after $t = \tau \Rightarrow I = 0.63\, I_0$.

→ <u>Half life</u> - $T = 0.693\tau = 0.693 \frac{L}{R}$.
$\left(I = \frac{I_0}{2}\right)$.

8. $\boxed{L_1 i_1 = L_2 i_2}$

9. <u>Decay of current</u> ⇒ $I = I_0 e^{-t/\tau}$

$\tau = L/R$. $\left(I_0 = \frac{\varepsilon}{R}\right) \Rightarrow$ at $t = 0$.

After $t = \tau$, $I = 0.37\, I_0$.

10. At $t = 0 \Rightarrow$ open circuit.
At $t = \infty \Rightarrow$ conducting wire.

11. <u>Mutual inductance</u> $\boxed{N_2\phi_2 = MI_1}$.

→ $\boxed{M_{12} = M_{21} = M}$ ⇒ Reciprocity theorem.

→ $\boxed{M = \frac{(\phi_T)_s}{I_p}}$ (M does not depend on I_p).

→ $\boxed{\varepsilon_2 = -M\left(\frac{dI_1}{dt}\right)}$.

→ <u>For two co-axial solenoids</u> - $\boxed{M = \mu_0 n_1 n_2 A l}$

→ <u>Two coplanar, co-axial, concentric, coils</u> -

$\boxed{M_{12} = \frac{\mu_0 n_1 n_2 A_2}{2R_1}}$ (2 = smaller loop, 1 = bigger loop) $R_1 >> R_2$.

→ $\boxed{M = K\sqrt{L_1 L_2}}$ K = coupling factor.
K = 1 for ideal/co-axial coupling.

→ $\boxed{K = \frac{\phi_s}{\phi_p}}$. ($K \leq 1$)

UNIT of L and M = Henry (H)
= Wb/A

→ Series $\boxed{L_{eq} = L_1 + L_2 \pm 2M}$ $+ \Rightarrow$ supporting

$- \Rightarrow$ opposing

→ Parallel $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots$

$$\boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}}$$

$+ \Rightarrow$ opposing

$- \Rightarrow$ supporting

12. Motional / Dynamic EMI - $\boxed{qE = qvB}$ $(E = vB)$

→ $\boxed{\varepsilon = El = Bvl}$ $\boxed{\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{l}}$

If any two vectors are $\parallel \Rightarrow \varepsilon = 0$.

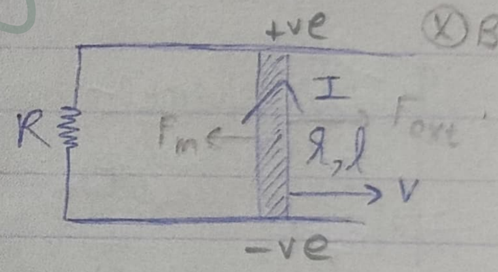
→ $\varepsilon = v_\perp Bl$ ($v_\perp = \vec{v} \perp$ to $\vec{l}$).

Direction of +ve terminal = direction of $\vec{F_m}$ on +ve charge q.

13. Motion of a straight Conductor In Horizontal Plane

→ $\varepsilon = Blv$

→ $I = \frac{\varepsilon}{R+r} = \frac{Blv}{R+r}$

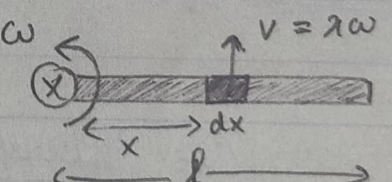


→ $F_m = IlB = \frac{B^2 l^2 v}{R+r} = F_{ext}$

→ $P_{ext} = \vec{F}_{ext} \cdot \vec{v} = \frac{B^2 l^2 v^2}{R+r} = (P_{thermal})_{\text{across resistance}} = I^2 R$.

14. EMF Due to Rotation

$d\varepsilon = B(dx)v = B(dx)(x\omega)$



$\boxed{\varepsilon = \frac{1}{2} B \omega l^2}$ l = length of rod from axis of rotation

= effective length for curved/irregular objects

→ $\varepsilon_{disc} = \frac{1}{2} B \omega r^2$ (like infinite rods in parallel).

15. Moving conductor rod in earth's magnetic field

(in module 5).

16 Periodic EMI - $\phi = \phi_0 \cos \omega t = NBA \cos \omega t$.

→ $e = e_0 \sin \omega t = NBA\omega \sin \omega t$.

→ $I = I_0 \sin \omega t = \frac{NBA\omega}{R} \sin \omega t = \frac{\varepsilon_0}{R} \sin \omega t = \frac{\phi_0 \omega}{R} \sin \omega t$.

17. DC/AC Generator/Dynamo - (same as periodic EMI)

→ slip ring ⇒ AC

split ring (commutator) ⇒ DC.

18. DC Motor electrical energy → mechanical energy

(opposite of generator).

e = back emf $= e_0 \sin \omega t$ $\quad \underline{E - e = iR.}$ $\quad \omega \uparrow \Rightarrow e \uparrow \Rightarrow i \downarrow$.

E = DC source $\quad e_0 = NBA\omega$.

→ $\boxed{\eta = \frac{e}{E} \times 100}$ $\quad \boxed{\eta_{maximum} = 50\%.}$

→ $e_{max} = E/2$. $\quad$ when $e = \frac{E}{2}$.

19. Transformer - no moving parts ∴ no mechanical loss in transfer ⇒ efficiency is higher than generator, motor.

→ $\boxed{\frac{\varepsilon_P}{\varepsilon_S} = \frac{N_P}{N_S}}$

For ideal transformer - $\boxed{\frac{\varepsilon_P}{\varepsilon_S} = \frac{I_S}{I_P}} = n$ or P $\quad (P_{in} = P_{out})$

n = turn ratio

P = transformation ratio.

For non-ideal transformer -

$$\boxed{\eta \% = \frac{P_S}{P_P} = \frac{\varepsilon_S I_S}{\varepsilon_P I_P} \times 100\%.}$$

→ Copper or Joule heating loss ($H = I^2 Rt$)

→ Flux leakage loss (due to air gap) ($K \neq 1$)

→ Iron losses — hysteresis losses (residual B)

— eddy current losses (due to varying B) (lamination of core)