

Basic Math

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Abscissa = x-axis
ordinate = y-axis.

1. $f(x) = x^2 + 1$
 $f(f(f(1))) = f(f(2)) = f(5) = 26.$

2. Quadratic eqⁿ $\Rightarrow ax^2 + bx + c = 0$
Roots: x_1, x_2 . $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots = $-\frac{b}{a}$

product of roots = $\frac{c}{a}$

Discriminant (D) = $b^2 - 4ac$

if $D > 0 \Rightarrow$ real and distinct roots

$D = 0 \Rightarrow$ real and equal roots

$D < 0 \Rightarrow$ imaginary roots.

3. Binomial Approximation

if $|x| \ll 1$

$\rightarrow (1+x)^n = 1 + nx$

$(1+x)^{-n} = 1 - nx$

$(1-x)^n = 1 - nx$

$(1-x)^{-n} = 1 + nx$

4. Logarithm

$\log m^n = n \log m$

$\ln m = \log_e m = 2.303 \log m$

$e \approx 2.7$

$\log mxn = \log m + \log n$

$\log \left(\frac{m}{n}\right) = \log m - \log n$

$\log_a a = 1$

$\log_a b = \frac{\log_c b}{\log_c a}$

$\log 1 = 0$

$\log 2 = 0.3010$

$\log 3 = 0.4771$

$\log 5 = 0.69$

$\log 10 = 1$

5. Componendo and dividendo

if $\frac{p}{a} = \frac{a}{b}$

Then by C&D $\Rightarrow \frac{p+q}{p-q} = \frac{a+b}{a-b}$

6. AP $n^{\text{th}} \text{ term} = t_n = [a + (n-1)d]$

$$S_n = \frac{n}{2} (a + l)$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

→ Sum of n natural no.s = $\frac{n(n+1)}{2}$

→ Sum of square of n natural no.s = $\frac{n(n+1)(2n+1)}{6}$

→ Sum of cube of n natural no.s = $\left[\frac{n(n+1)}{2} \right]^2$

7. GP

$$n^{\text{th}} \text{ term} = t_n = a \cdot r^{(n-1)}$$

$$r = \text{common ratio} = \frac{\text{next term}}{\text{previous term}}$$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

$$S_{\infty} = \frac{a}{1-r}$$

8. $\pi = \frac{22}{7} = 3.14$

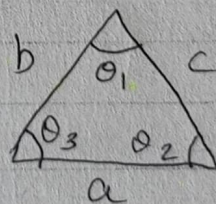
$$\pi^2 = 10$$

$$\frac{1}{\pi} = 0.3$$

9. Trigonometry - $\theta (\text{radian}) = \frac{\text{arc length}}{\text{radius}}$

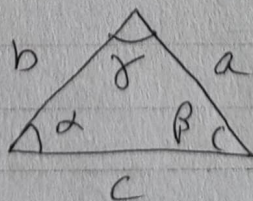
① Cosine rule for triangle -

$$c^2 = a^2 + b^2 - 2ab \cos \theta_3$$



② Sine rule for triangle

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$



③ If $y = a \sin \theta + b \cos \theta$

$$y_{\max} = \sqrt{a^2 + b^2}$$

$$y_{\min} = -\sqrt{a^2 + b^2}$$

④ If θ is very small ($\theta \leq 5^\circ$)

$$\sin \theta \approx \tan \theta \approx \theta \text{ (rad.)}$$

$$\cos \theta \approx 1.$$

10. Geometry -

Distance formula -

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\rightarrow \text{slope} = m = \tan \theta$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$\rightarrow y = mx + c$$

$$(y - y_1) = m(x - x_1)$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\swarrow$ $\searrow$
 x -intercept y -intercept

11. Differentiation

$$f'(x) = \frac{d(f(x))}{dx} = \tan \theta = \text{slope} = m$$

$$\rightarrow \frac{d(x^n)}{dx} = nx^{n-1}$$

$$\frac{d(ky)}{dx} = k \frac{dy}{dx}$$

$$\rightarrow \frac{d(\text{constant})}{dx} = 0$$

$$\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\rightarrow \frac{d(u \cdot v)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\rightarrow \frac{d(u/v)}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

y	dy/dx
$\log_e x$	$\frac{1}{x}$
e^x	e^x
e^{ax}	ae^{ax}

y	dy/dx
$\sin \theta$	$\cos \theta$
$\cos \theta$	$-\sin \theta$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cdot \cot x$
$\sec x$	$\sec x \cdot \tan x$
$\tan x$	$\sec^2 x$

→ Maxima and Minima

$$\frac{d^2y}{dx^2} < 0$$

$$\frac{d^2y}{dx^2} > 0$$

$\frac{dy}{dx} = 0 \rightarrow$ point of maxima or minima.

12. Integration (Anti-derivative)

$f(x)$	$\int f(x) dx$
x^n	$\frac{x^{n+1}}{n+1}$
dx	x
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\sec^2 x$	$\tan x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\frac{1}{x}$	$\log_e x$
e^x	e^x
$\frac{1}{ax+b}$	$\frac{1}{a} \ln(ax+b)$
e^{ax}	$\frac{1}{a} e^{ax}$
$\sin(Ax+b)$	$-\frac{\cos(Ax+b)}{A}$
$\cos(Ax+b)$	$\frac{\sin(Ax+b)}{A}$

• Remember integration by substitution.

• Definite integration

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

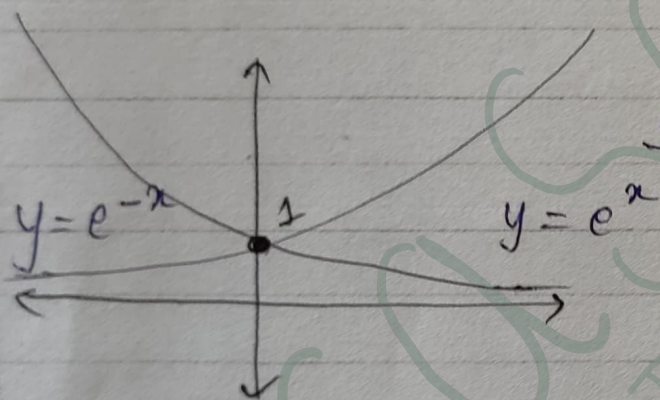
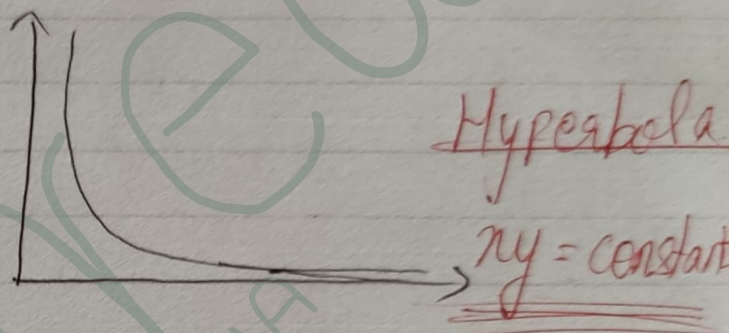
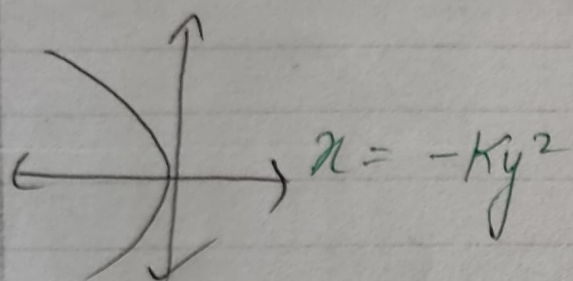
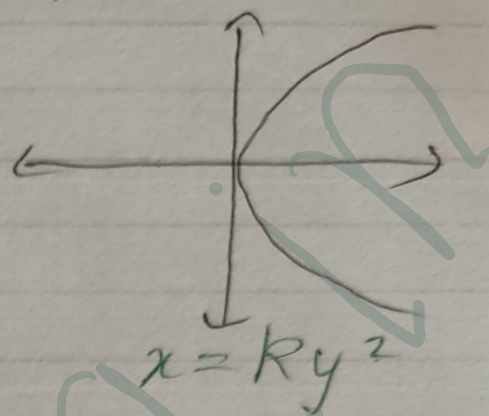
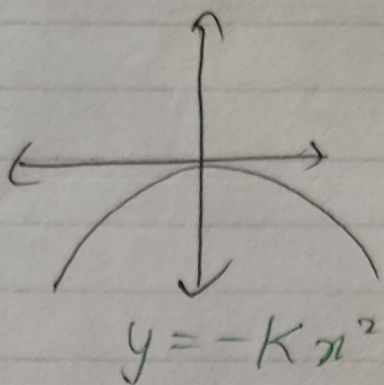
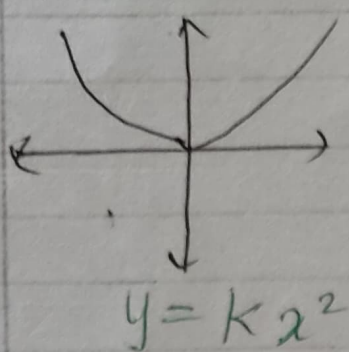
• Avg. Value

$$y = f(x)$$

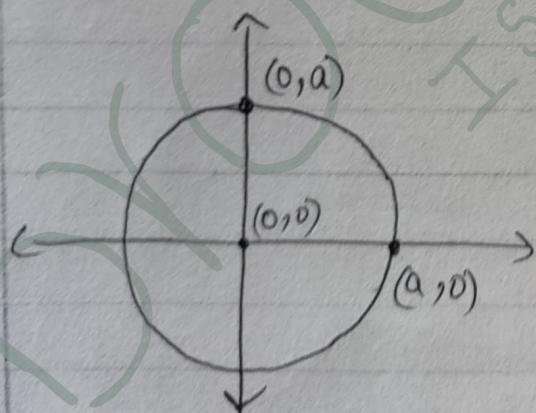
$$\langle y \rangle = \bar{y} = y_{\text{avg.}}$$

$$\langle y \rangle = \frac{\int_a^b f(x) dx}{\int_a^b dx}$$

Area under curve/graph can be found using integration.

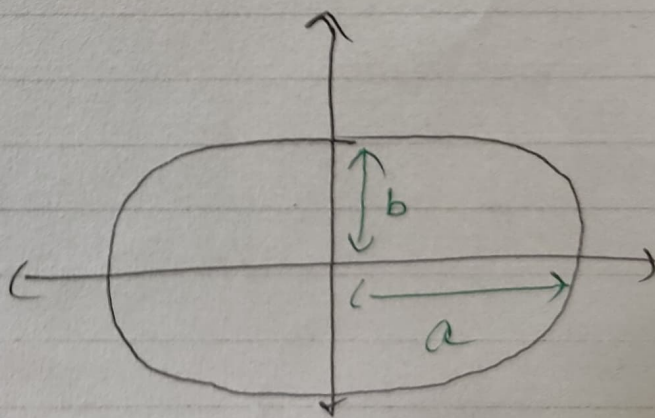
13. Graphs -

Exponential graph



Circle

$$x^2 + y^2 = a^2$$



Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$